1.

CE 8402 - Strength of Material II

Unit :- I Energy Principles:

Strain energy:

If a body is strained due to loading energy is absorbed in the body.

The energy which is absorbed in the body due to straining effect is known as strain energy.

SI Unit - Nm or J.

Strain energy density:

The strain energy density of the material is defined as the strain energy per unit volume.

$$\text{Strain energy density} = \frac{U}{V}$$

Strain energy due to Axial Load (Tension and Compression)

$$\left.\begin{array}{l}\text{Strain energy stored in the} \\ \text{bar } U\end{array}\right\} = \frac{1}{2} \times W \times \delta L$$

$$\text{Strain energy, } V = \frac{\sigma^2 V}{2E}$$

$$\text{Modulus of resistance} = \frac{\text{proof resilience}}{\text{Volume of the body.}} = \frac{\sigma p^2}{2E}$$

Graudally applied Load:-

Strain energy stored in the body = $\frac{1}{2} \times \sigma_A \times \delta L$

Strain energy = work done

$$\frac{1}{2} \sigma_A \times \delta_L = \frac{1}{2} \times W \times \delta L$$

$$\sigma = \frac{2W}{A}$$

Impact Loadings:-

External work done = Energy stored in bar

Work done = $W(h + \delta L)$

Impact factor, $n = 1 + n'$

$$n' = \sqrt{1 + \frac{2h}{\delta L}}$$

problem: :- A steel bar 5 cm by 5 cm in section, 4m long is subjected to an axial pull of 130 kN. Taking E = 200 GN/m² Calculate the alternation in the length of the bar. Calculate also the amount of energy stored in the bar during the extension

Given:

$E = 200\ GN/m^2 = 200 \times 10^9\ N/m^2$

$l = 4m$, $A = 5 \times 5 = 25 cm^2 = 25 \times 10^{-4} m^2$

$W = 130\ kN = 130 \times 10^3 N$

2.

Solution :-

(i) Change in Length, $\delta L = \frac{WL}{AE}$

$$= \frac{130 \times 10^3 \times 4}{25 \times 10^{-4} \times 200 \times 10^9}$$

$$= 0.00104 \text{ m} = 1.04 \text{ mm.}$$

(ii) Amount of energy stored

$$U = \frac{\sigma^2}{2E} \times AL$$

$$\sigma = W/A = \frac{130 \times 10^3}{25 \times 10^{-4}} = 52000000$$

$$U = \frac{(52 \times 10^6)^2}{2 \times 200 \times 10^9} \times 25 \times 10^{-4} \times 4.$$

$$U = 67.6 \text{ Nm (or) J.}$$

Strain energy due to Bending :-

Strain energy $U = \frac{1}{2} \times W \times y_c$

where y_c = deflection, W = Load.

Strain energy due to shear

Strain energy $U = \frac{\tau^2}{2C} \times$ Volume of block.

where

$\tau \Rightarrow$ shear stress

C = Modulus of rigidity.

Strain energy in torsion:

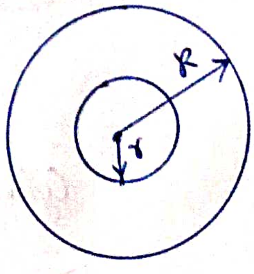
For solid shaft:

$$U = \frac{\tau^2}{4C} \times \frac{R^2 + r^2}{R^2} \times \text{Volume.}$$

problem: The external diameter of hollow shaft is twice the internal diameter. It is subjected to pure torque and it attains. A maximum shear stress τ, show that the strain energy stored per unit volume of the shaft is $\frac{5\tau^2}{16C}$ such a shaft is required to transmit 5400 kw at 110 rpm with uniform torque. the maximum stress not exceeding 84 MN/m^2. Determine

1) Diameter of the shaft 2) Energy stored. per m^3

Given:



$C = 90\ GN/m^2$; $\tau = 84\ MN/m^2$

power = 5400 kw ; 5400 × 1000 W

$N = 110$ rpm $= \frac{110}{60}$ rps

$$U \text{ (hollow shaft)} = \frac{\tau^2}{4C} \times \left(\frac{R^2 + r^2}{R^2}\right)$$

$$= \frac{\tau^2}{4CR^2} \times \left(R^2 + \frac{R^2}{u}\right)$$

$$\frac{U}{\text{Volume}} = \frac{5\tau^2}{16C}$$

$$\tau = \frac{P}{2\pi N} = \frac{5400 \times 1000 \times 60}{2 \times \pi \times 110}$$

$$= 468783 \text{ Nm}$$

$$T = I_p \frac{\tau}{R} = \frac{\pi}{32} \times \left(D^4 - d^4 \times \frac{\tau}{(D/2)}\right)$$

$$468783 = \pi/16 \times \left(\frac{D^4 - d^4}{D}\right)\tau$$

$$D = 0.312 \text{ m} = 312 \text{ mm}$$

$$d = 156 \text{ mm.}$$

Energy stored

$$\frac{U}{\text{Volume}} = \frac{5}{16} \times \frac{\tau^2}{C}$$

$$= 24.5 \text{ kJ/m}^3$$

Castigliano theorem:-

Theorem :- I :- The partial derivative of strain energy divided by partial derivative of force gives the deflection

$$\therefore \frac{\partial U}{\partial W} = \delta \text{ (deflection)}$$

Theorem - II :- The partial derivative of deflection divided by partial derivative of moment of gives the rotation $\frac{\partial \theta}{\partial m} = \theta$

uses:-

1.) To determine the displacement of complicated structure

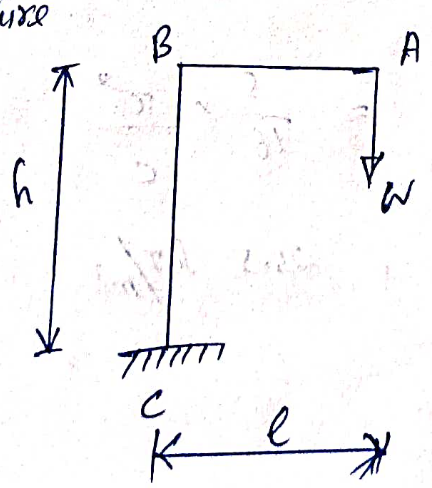
2) To find the deflection of curved beam, spring etc.,

point to remember:-

* Treat all the load and couples as variable and carried out the partial differentiation.

* To find out the deflection apply dummy V load W at the point and give value zero at the end.

problem :- Find the strain energy and deflection at A for the given structure



Solution:- Take moment between BA, CB.

$M_{x_1} = Wx \qquad 0 < x < l$

$M_{x_2} = Wl \qquad 0 < x < h$

$$M_1^2 = W^2 x^2 \qquad M_{x_2} = W^2 L^2$$

$$U = \int_0^l \frac{M^2\,dx}{2EI}$$

$$U_1 = \frac{1}{2EI} W^2 \int_0^l x^2\,dx.$$

$$= \frac{1}{2EI} \times \frac{W^2 l^3}{3}$$

$$U_1 = \frac{1}{6EI} \times W^2 l^3$$

$$U_2 = \frac{1}{2EI} W^2 l^2 \int_0^h dx = \frac{1}{2EI} W^2 l^2 (x)_0^h$$

$$= \frac{W^2 l^2}{6EI}(l + 3h)$$

$$\delta = \frac{W l^2}{3EI}(l + 3h)$$

Maxwell's reciprocal.

The work done by the 1^{st} system of load due to displacement caused by second system of load equations.

The work done by second system of load due to displacement caused by the 1^{st} system of load.

$$U_{A,B_e} = U_{B,A}$$

$$\sum_{i=1}^{n} (P_i)_A (\delta_i)_B = \sum_{j=1}^{m} (P_j)_B (\delta_j)_A$$

principle of virtual work (or) Unit Load method.

⇒ The principle of virtual work was developed of Johann Bernoulli in year 1717.

⇒ This is the most versatile of the method available for computing deflection of structure.

⇒ This method also termed as Unit Load method A virtual force acting through a real displacement.

⇒ The principle of virtual work is based on the conservation of energy for a structure. which implies that work done on a structure.

work of external Loads = Work of internal forces.

Virtual work equation for truss:-

⇒ Apply the equation of virtual work to determine the desired displacement.

$k \rightarrow$ virtual force $\qquad F \rightarrow$ Real force.

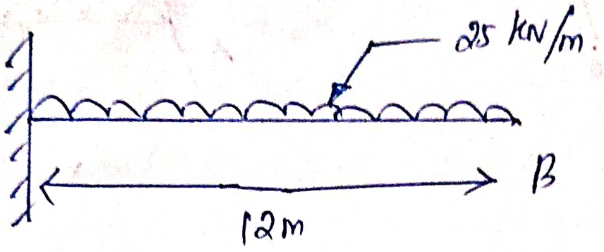
$$\Delta = \sum \frac{kFL}{AE}$$

Where $k \rightarrow$ Internal force = Virtual force.

$F \rightarrow$ Load acting on the member.

5.

Problem: using the method of virtual work determine the vertical displacement point of the beam. Take $E = 2\times10^5$ Mpa, $I = 825\times10^7$ mm^4



Given:

$E = 2\times10^5$ Mpa $= 2\times10^{11}$ N/m^2

$I = 825\times10^7 mm^4 = 825\times10^{-5} m^4$

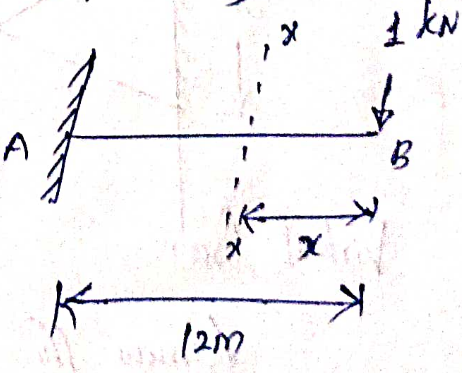
$$1A = \int m \frac{M dx}{EI}$$

Solution:

1) Remove all real loads.

2) Apply Unit Load at point B

Taking moment between AB (Consider right side)



Virtual Moment, $M_x = -1\times x$ $0 < x < 12$

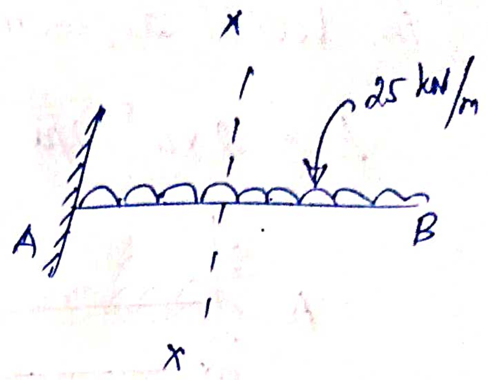
Consider Actual beam

Real Moment, $M_x = -25\frac{x^2}{2}$

$= 12.5x^2$

Virtual equation $= \int_0^l m \frac{M dx}{EI}$

$$= \int_0^{12} \frac{(-1x)(-12.5x^2)\,dx}{EI}$$



$$= \frac{12.5}{EI}\left[\frac{x^4}{4}\right]_0^{12}$$

$$= \frac{64800}{EI}$$

$$= \frac{64800 \times 10^3}{2\times10^8 \times 825\times10^{-5}} = 0.0393\,m \text{ (or) } 39.3\,mm.$$

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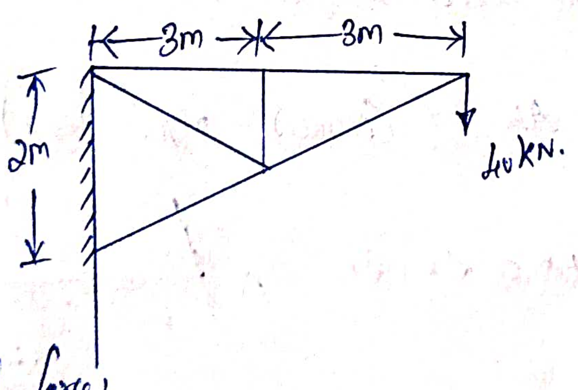
problem: Determine the vertical deflection at the free end of the Cantilever truss shown in figure. Take Cross sectional area of Compression member 850 mm² and tension member as 1000 mm². Modulus of elasticity E = 210 GPa for all the member.



Virtual force:

Remove the external load and apply Unit vertical Load at free end.

Joint A

$\Sigma v = 0$ In Joint 'A' Assume all are tension

6.

To find angle:

$$\Sigma V = 0$$

for $\frac{6m}{2} = \frac{3}{x}$

$$x = \frac{3 \times 2}{6}$$

$$x = 1m$$

$\Sigma H = 0.$

$$K_{AC} \sin\theta + 1 = 0$$

$$K_{AC} = \frac{-1}{\sin (18.43)}$$ $K_{AC} = -3.16$

$$K_{AB} + K_{AC} \cos (18.43) = 0$$

$$K_{AB} = +3 \text{ kN}$$

$K_{AC} = -3.16$; $K_{AB} = 3$

$\tan\theta = 1/3$; $\theta = 18°26'5'' = 18.43°$

Joint B

$\Sigma V = 0$, $\Sigma H = 0$

$$K_{BD} - K_{AB} = 0$$

$$K_{BD} = K_{AB}$$

$$= 3$$

$$K_{BC} = 0 = K_{BD}$$

$\Sigma V = 0$

$K_{AC} \sin 18.43 + K_{CD} \sin 18.43 - K_{EC} \sin 18.43 = 0$

$K_{AC} = -3.16$

$\sin 18.43 = 0.316$, $\cos 18.43 = 0.948$

$-3.16 \times 0.316 + k_{CD}\, 0.316 - k_{EC}\, 0.316 = 0$

$0.316\, k_{CD} - 0.316\, k_{EC} = 1$

$\Sigma_H = 0$

$K_{CD} \cos 18.43 + K_{EC} \cos 18.43 - k_{AC} \cos 18.43 = 0$

$0.948\, k_{CD} + 0.948\, K_{EC} = -3$

Solve ① & ②

$K_{CD} = 0$; $k_{EC} = 3.16$

S.No	Member	K	F	L	ΣFFL
1	AB	3	120	3	1080 (T)
2	AC	-3.16	-126.4	3.16	1262.18 (C)
3	BC	0	0	-	-
4	BD	3	120	3	1080 (T)
5	CD	0	0	-	-
6	CE	-3.16	-126.4	3.16	1262.17 (C)

Total = 2160 (T) & Comp = 25243.

7.

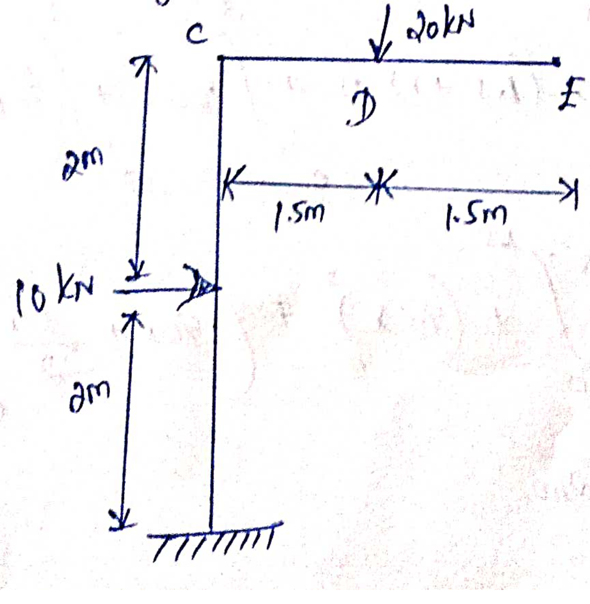
Lack of fit in truss:

Space truss with a high degree of static redundancy, such as double layer Grid, are highly ~~sentitel~~ sensitive to imperfection. substantial reduction in load bearing Capacity are caused primarily by random lack of fit of member, associated with brittle - type Compressive member buckling.

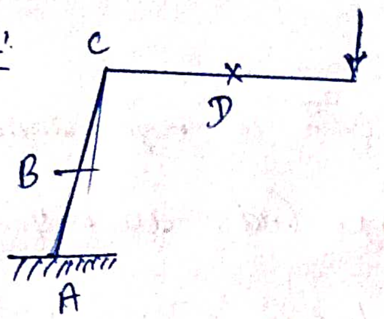
Thermal Effect:

In building design, thermal mass is a property of the mass of a building which enables it to store heat, providing "interia" against temperature fluctuations it is sometimes known as the thermal (or) temperature effect.

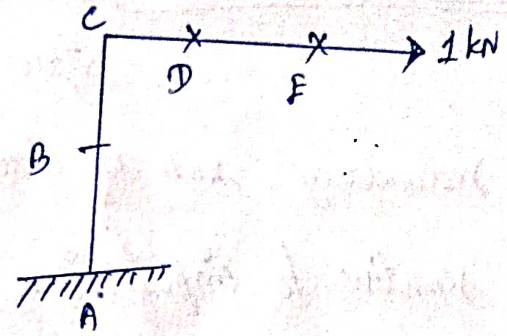
Problem: Determine the vertical and the horizontal deflection at the free end of the bent shown figure. Assume uniform flexural rigidity. EI throughout.



Solution:



Frame with unit vertical load @ E



portion	ED	EC	CB	BA
Origin	E	D	C	B
limit	0 to 1.5	0 to 15	0 to 2	0 to 2
M	0	$-20x$	-30	$-30-10x$
M_1	x	$-(1.5+x)$	-3	-3
M_2	0	0	$-x$	$-(x+2)$
Flexural rigidity	EI	EI	EI	EI

$$EI\,\Delta_{EV} = \int M M_1\,dx$$

$$= 0 + \int_0^{1.5} 20x\,(1.5+x)\,dx + \int_0^{2} 90\,dx + \int_0^{2} (90+30x)\,dx$$

$$= \left[\frac{30x^2}{2} + \frac{20x^3}{3}\right]_0^{1.5} + \left(90x\right)_0^{2} + \left(90x + \frac{30x^2}{2}\right)_0^{2}$$

$= 56.25 + 180 + 240.$

3.

$$\Delta_{Ev} = \frac{476.25}{EI}$$

$$EI\,\Delta_{EH} = \int M m_2\, dx$$

$$= 0 + 0 + \int_0^2 30x\, dx + \int_0^2 (30 + 10x)(x+2)\, dx.$$

$$= \left(15x^2\right)_0^2 + \int_0^2 (10x^2 + 50x + 60)\, dx.$$

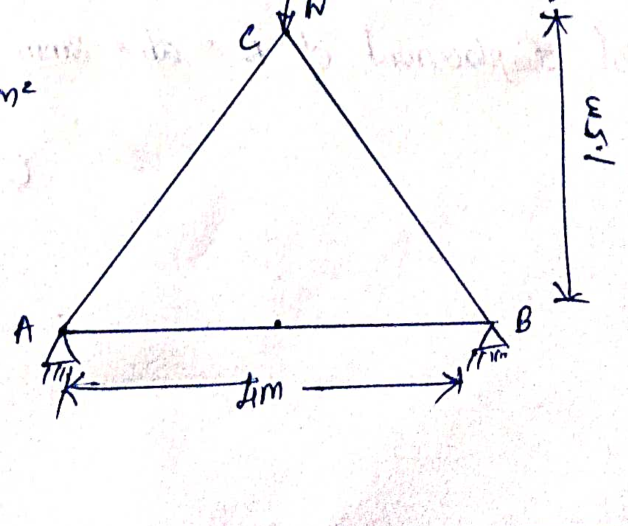
$$= 60x + \left(\frac{10x^3}{3} + 50 \times \frac{x^2}{2} + 60x\right)_0^2$$

$$= 306.67$$

$$\Delta_{EH} = \frac{306.67}{EI}$$

Willot Mohr's diagram.

Problem:- Find the displacement of the point B and C of the truss shown in figure. The areas of the member are such that that Compression member are subjected to a stress of 100 N/mm² and the tension member are subjected to a stress of 150 N/mm² Take $E = 2 \times 10^5$ N/mm²



Solution :-

Length $AC = BC = \sqrt{2^2 + 1.5^2}$

Decrease in length of AC (or) CB

$$= \delta L_1 = \delta L_2 = \frac{P_1}{E} \times L_1$$

$$= \frac{100 \times 2500}{2 \times 10^5} = 1.25\,mm$$

Increase in length of AB

$$\delta L_3 = \frac{150 \times 4000}{2 \times 10^5} = 3\,mm$$

Take convenient point a and draw ab

$$= \delta L_3 = 3\,mm$$

Draw ac_1 parallel to AC and making $ac_1 = \delta L_1 = 1.25\,mm$

Draw bc_1 parallel to BC and making $bc_1 = \delta L_2 = 1.25\,mm$

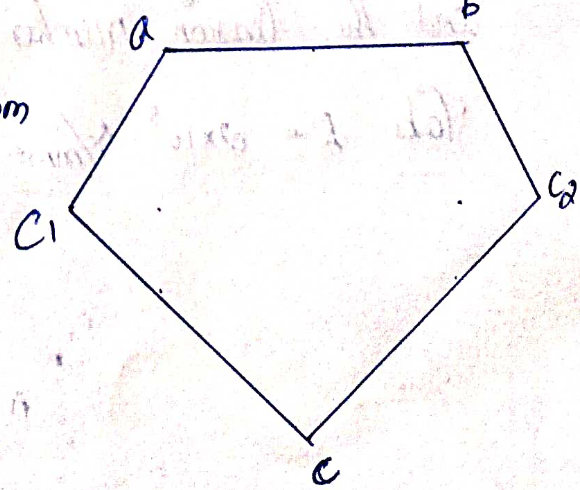
Draw $c_1 c_2$ perpendicular to ac_1

Draw $c_3 c$ perpendicular to bc_2 thus obtain the point c

Now actual displacement of vertex

$C = ac = 3.60\,mm$

Actual displacement of $B = ab = 3\,mm$



1

Unit: II Indeterminate Beam.

Concept of analysis:

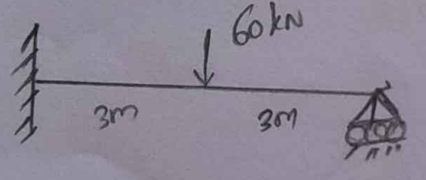
Any structure which cannot be solved by means of the three static equilibrium condition, then structure called indeterminate (or) redundant structure.

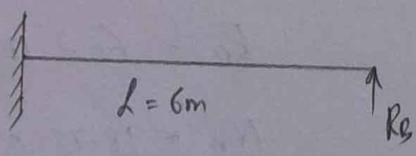
propped Cantilever beam:

A cantilever supported at free end using prop (or) support is known as propped cantilever beam.

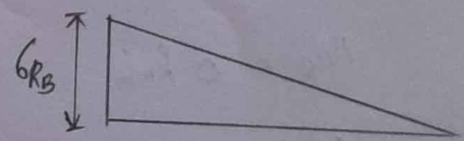
problem:

Draw SFD and BMD for the propped cantilever beam loaded AB shown in fig. Draw BMD for the prop.





Solution:



$x_1 = 2/3 \times 1 = 4m$

Deflection due to prop.

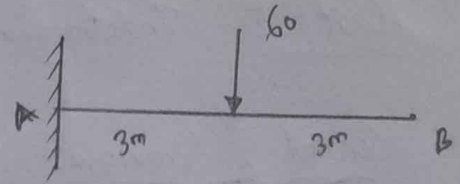
$$y_B' = \frac{A_1\bar{x}_1}{EI} = \frac{18 R_B \times 4}{EI}$$

$$= \frac{72 R_B}{EI}$$

$A_1 = \frac{1}{2} \times b \times h = \frac{1}{2} \times 6 R_B \times 6$

$A_1 = 18 R_B$

Draw a BMD for Cantilever beam

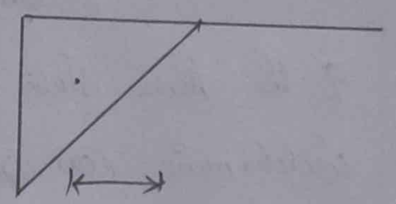


$A_2 = \frac{1}{2} \times b \times h = \frac{1}{2} \times 180 \times 3$

$= 270\ kNm^2$

$\bar{x}_2 = 5m$

Downward deflection due to Load.



$$y_B = \frac{A_2 \bar{x}_2}{EI} = \frac{270 \times 5}{EI} = \frac{1350}{EI}$$

$y_B' = y_B$

$$\frac{72 R_B}{EI} = \frac{1350}{EI}$$

$$R_B = \frac{1350}{72} = 18.75\ kN.$$

$R_A + R_B = 60$

$R_A = 60 - 18.75 = 41.25\ kN$

$M_A = 18.75 \times 6 - 60 \times 3 = -67.5\ kNm$

$M_B = 0\ kNm$

$M_C = R_B \times 3 = 18.75 \times 3 = 56.25\ kNm.$

SFD

Start from force end B

Consider left of B = 0

Right of B = 18.75 kN

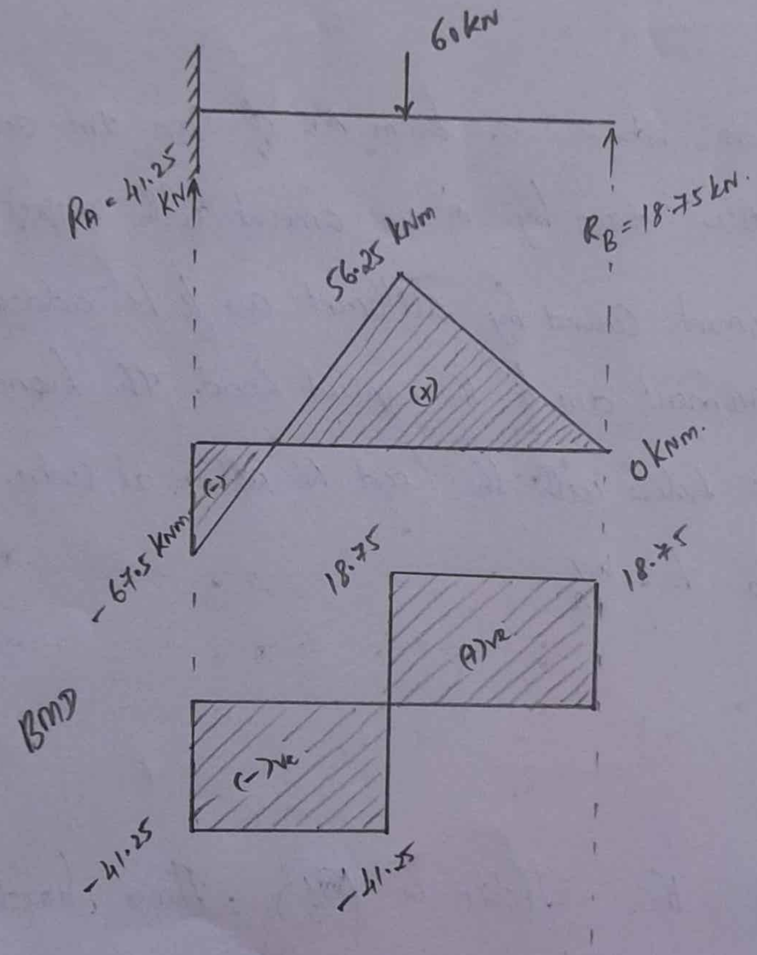
Left of C = 18.75 kN

2.

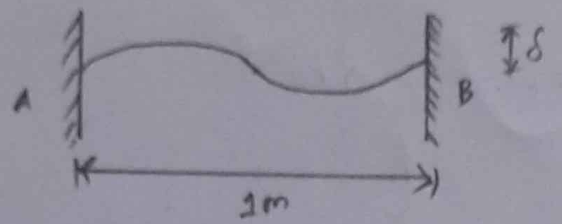
Right of C = 18.75 − 60 = −41.25 kN

Left of A = −41.25 kN

Right of A = −41.25 + 41.25 = 0 kN

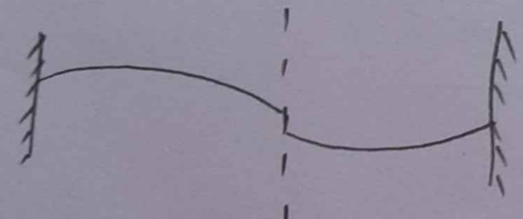


Effect of sinking of supports (Rotation of support.)



The above figure shows a beam AB of span 1m and the support B settles down by A an amount with respect A. The difference moment caused by settlement are to be added in the fixed end moment due to the applied Load. The beam is split into two halves with the Load W acting at centre.

Support reaction to be $W/2$



For each half the deflection is $(\delta/2)$. Hence fixed end moment is $(W \times 1/2)$ at each end.

$$\frac{\delta}{2} = \frac{WL^3}{3EI} \qquad L = 1/2$$

$$\delta/2 = \frac{W(1/2)^2}{3EI} \qquad W = \frac{12EI\delta}{2}$$

$$M = W(1/2) = \frac{12EI\delta}{L^2} \times \left(\frac{\delta}{2}\right)$$

$$= \frac{6EI\delta}{L^2}$$

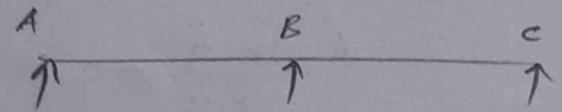
3

If support B lowers by δ $M_A = M_B = \dfrac{-6EI\delta}{l^2}$

If support B lower by δ $M_B = M_B = \dfrac{6EI\delta}{l^2}$

If "B" lowers $R_A = \dfrac{12EI\delta}{l^3}$, $R_B = -\dfrac{12EI\delta}{l^2}$

Theorem of three-moment:-



$$\frac{M_A L_1}{E_1 I_1} + 2M_B\left[\frac{l_1}{E_1 I_1} + \frac{l_2}{E_2 I_2}\right] + \frac{M_C l_2}{E_2 I_2} + \frac{6a_1\bar{x}_1}{l_1 E_1 I_1} +$$

$$+ \frac{6a_2\bar{x}_2}{l_2 E_2 I_2} = \frac{6\delta_1}{l_1} + \frac{6\delta_2}{l_2}$$

L_1 = Length of span AB

L_2 = Length of span BC

$a_1\bar{x}_1$ = First moment of BMD for span AB considering the origin at A.

$a_2\bar{x}_2$ = First moment of BMD for span BC, considering the origin at C.

E_1I_1 = Flexural rigidity for the span AB

E_2I_2 = Flexural rigidity for the span BC

δ_1 = sinking of the support A w.r.t of support B

δ_2 = sinking of the support C w.r.t of support B

Case (i)

$$\delta_1 = \delta_2 = 0$$

$$\frac{M_A l_1}{E_1 I_1} + 2M_B\left[\frac{l_1}{E_1 I_1} + \frac{l_2}{E_2 I_2}\right] + M_C \frac{l_2}{E_2 I_2} + \frac{6a_1 \bar{x}_1}{l_1 E_1 I_1}$$

$$+ \frac{6a_2 \bar{x}_2}{l_2 E_2 I_2} = 0$$

Case (ii) Flexural rigidity same.

$$E_1 I_1 = E_2 I_2 ; \quad \delta_1 = \delta_2 = 0$$

$$M_A l_1 + 2M_B(l_1 + l_2) + M_C l_2 + \frac{6a_1 \bar{x}_1}{l_1} + \frac{6a_2 \bar{x}_2}{l_2} = 0$$

Continuous Beam:-

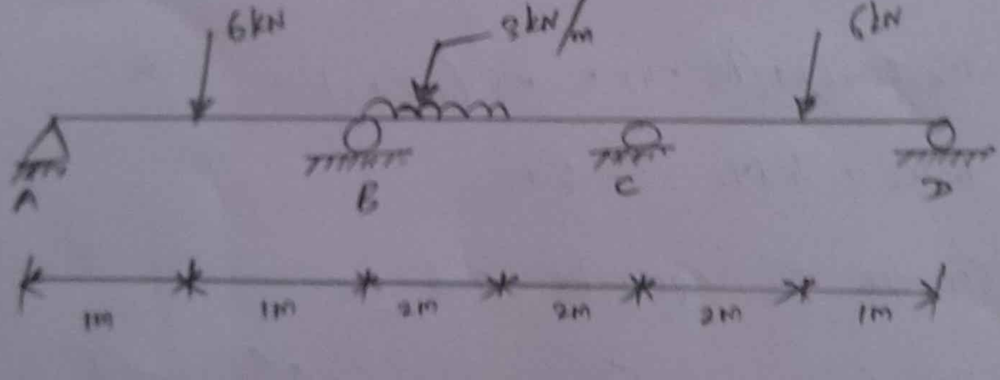
A beam is generally supported on a hinge at one end and a roller bearing at the other end.

The reaction are determined by using static equilibrium equation, such as known as statically determined structure.

If the end of the beam are restrained (clamped/encastred) fixed, then the moment make the structural element to be statically indeterminate structure.

A continuous beam is one being more than the one span & it is carried by several support (minimum three).

Problem:- A continuous beam ABCD is figure. Draw SFD and BMD indicating the salient point.



Soln:- We know that.

Simply supported moment are

$$\text{Span } AB = \frac{Wl}{4} = \frac{6 \times 2}{4} = 3$$

Span BC

$$\frac{WL^2}{8} = \frac{3\times4^2}{8} = 6.$$

Span CD

$$\frac{Wab}{l} = \frac{6\times2\times1}{3} = 4.$$

Consider span AB and BC

$$M_A l_1 + 2 M_B (l_1+l_2) + M_C l_2 = -6\left[\frac{a_1 x_1}{l_1} + \frac{a_2 x_2}{l_2}\right]$$

$$a_1 = \tfrac{1}{2}\times b\times h = \tfrac{1}{2}\times2\times3 = 3$$

$$a_2 = \tfrac{2}{3}\times4\times6 = 16$$

$x_1 = 1m$, $x_2 = 2m$

$l_1 = 2m$, $l_2 = 4m$

$$\frac{a_1 x_1}{l_1} = \frac{3\times1}{2} = 1.5 \quad / \quad \frac{a_2 x_2}{l_2} = \frac{16\times2}{4} = 8$$

$$2M_B(2+4) + (M_C\times4) = -6(1.5+8)$$

$$12M_B + 4M_C = -57$$

Consider the span BC and CD

$$M_B l_1 + 2M_C(l_1+l_2) + \cancel{M_D l_2}^{\,0} = -6\left[\frac{a_1x_1}{l_1} + \frac{a_2x_2}{l_2}\right]$$

$$4M_B + 2M_C(4+3) = -6\left(\frac{a_1x_1}{l_1} + \frac{a_2x_2}{l_2}\right)$$

$$a_2 = \tfrac{1}{2}\times3\times4 = 6$$

5.

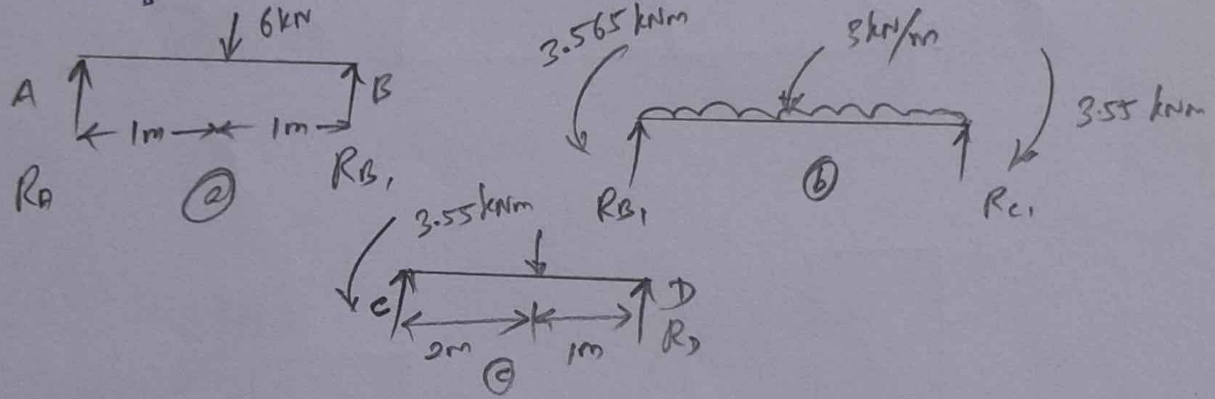
$$\frac{a_1 x_1}{l_1} = 8. \quad / \quad \frac{a_2 x_2}{l_2} = \frac{6 \times 1.33}{3} = 2.667.$$

$$4M_B + 14M_c = -6(8 + 2.667)$$

$$4M_B + 14M_c = -64.$$

Solve ① and ②

$M_B = -3.655$ $\qquad M_c = -3.55$ kNm



ⓐ $R_{B_1} \times 2 - 6 \times 1 - 3.566 = 0.$

$R_{B1} = 4.73$ kN

$R_A = 1.22$ kN

ⓑ $3.566 = R_{c_1} \times 4 - \frac{3 \times 4^2}{2} - 3.55$

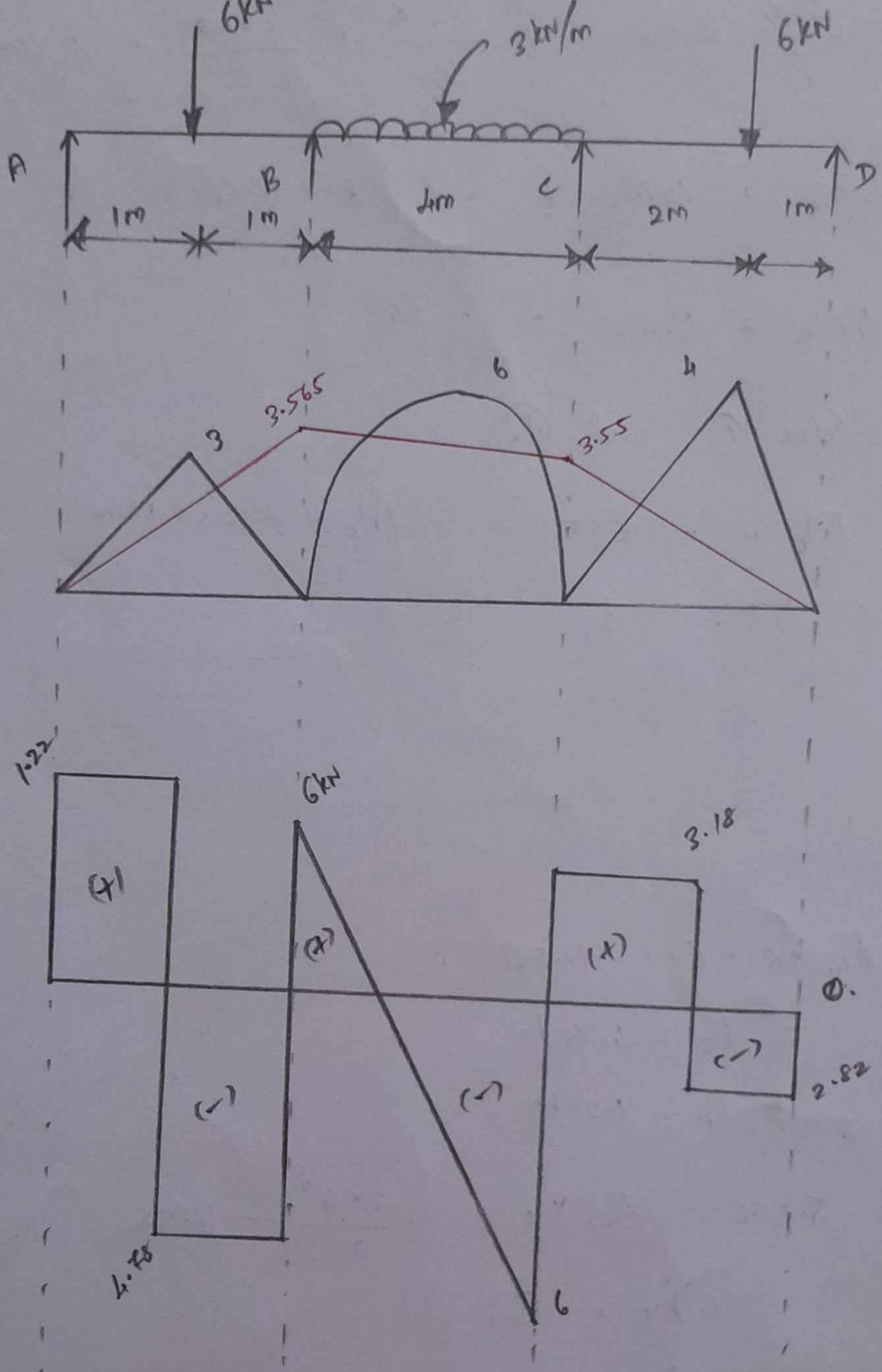
$6 \text{ kN} = R_{c_1}$

$R_{B_1} = 6$ kN

$R_D \times 3 - 6 \times 2 = 0$

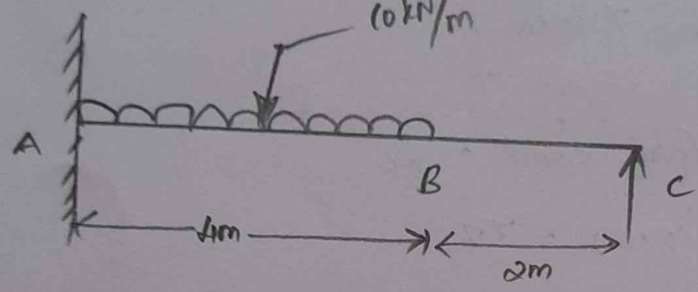
$R_D = 4$ kN

$R_{c_2} = 6 - 4 = 2$ kN


6kN
3kN/m
6kN
A
B
C
D
1m
1m
4m
2m
1m
3
3.565
6
3.55
4
1.22
(+)
(-)
4.78
6kN
(+)
(-)
6
3.18
(+)
0.
(-)
2.82

6.

Problem: A propped cantilever beam ABC of span 6m fixed at A and propped at C is loaded with an UDL of 10 kN/m for a length of 4m from the fixed end. Find the prop reaction. Draw shear force and bending moment diagram. Find the maximum sagging B.M and point of contraflexure.



$$\delta_{cp} = \delta_c$$

Deflection due to prop $= \dfrac{A_1 \bar{x}}{EI}$

$$A_1 = \frac{1}{2} \times 6 \times 6.$$

$$= \frac{1}{2} \times 6 \times 6 R_c$$

$$\bar{x} = 4m$$

$$= \frac{18 R_c \times 4}{EI}$$

$$\delta_{cp} = \frac{72 R_c}{EI}$$

Deflection due to Load $= \dfrac{A \bar{x}}{EI}$

$A_2 = \frac{1}{2} \times b \times h$

$= \frac{1}{2} \times 4 \times 80$

$= 106.667\ m^2$

$$\frac{72 R_c}{EI} = \frac{533.33}{EI}$$

$72 R_c = 533.33$

$\boxed{R_c = 7.40\ kN}$

$R_A = \text{Total Load} - R_c$

$= (10 \times 4) - 7.4$

$= 32.6\ kN$

Shear force Diagram.

Shear force at left of A = 0

Shear force at right of B = $32.6 - (10 \times 4)$

$= -7.4\ kN$

Shear force at Right of B = $-7.4\ kN$

Shear force at left of C = $-7.4\ kN$

Right of C = 0.

BMD

B.M at C = 0 kNm

BM at B = $7.4 \times 2 = 14.8\ kNm$

BM at A = $7.4 \times 6 - 10 \times \frac{4^2}{2} = -35.6\ kNm$

7.

To find the max. BM (S.F = 0)

S.F at E = 0

x distance from A

Find shear force at E = $32.6 - \frac{10x^2}{2}$

$-5x^2 = -32.6$

$x^2 = 6.52$; $x = 2.55\,m$

Moment at E = $7.4 \times (2 + 1.45) - \frac{10 \times 1.45^2}{2}$

Consider Right side.

$= 25.53 - 10.512 = 15.0\ kNm$

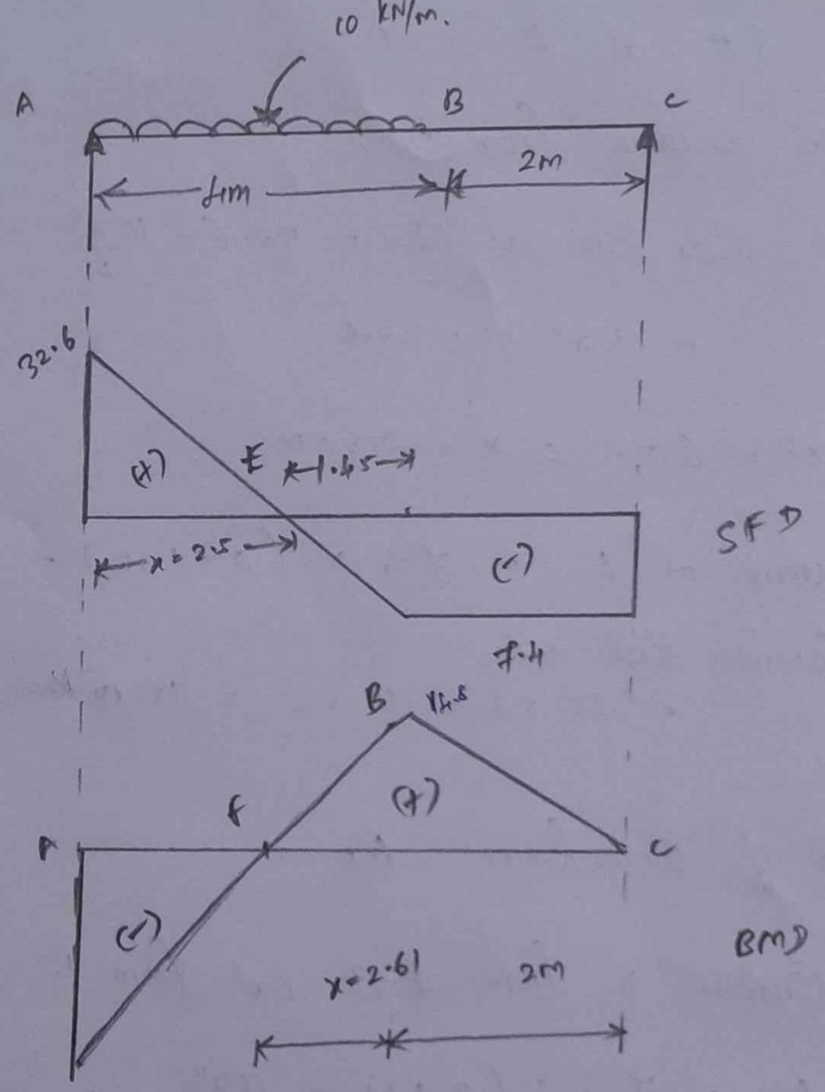
Point of Contraflexure $M_F = 0$

Consider 'x' from Right end from B

$M_F = 7.4 \times (2 + x) - \frac{10x^2}{2} = 0$

$14.8 + 7.4x - 5x^2 = 0$

$y = 2.61\,m$ from B.

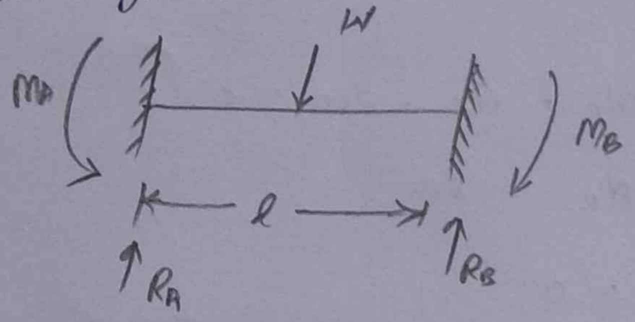


10 kN/m.
A
B
C
4m
2m
32.6
(+)
E
1.45
x = 2.5
(-)
7.4
SFD
B
14.8
F
(+)
A
C
(-)
x = 2.61
2m
BMD

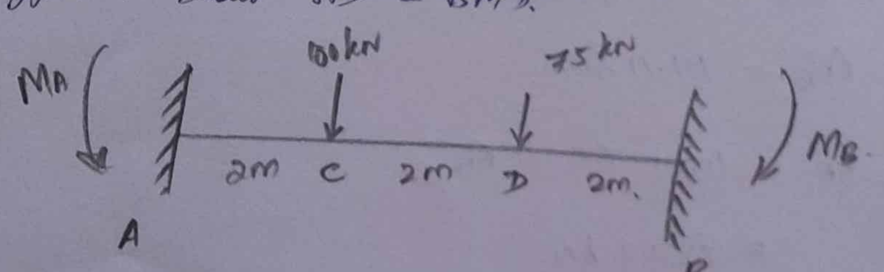
8.

Fixed Beam:

A fixed beam is a beam the ends of which are constrained (or) built into remain in horizontal position.



Problem: A fixed Beam of 6m span carries point Load of 100KN and 75kN as shown in fig. find the following (i) finding moment at end. (ii) Reaction at the support. Draw SFD & BMD.



Solution:

$$A_1 = \frac{1}{2} \times 6R_B \times 6 = 18 R_B$$

$$\bar{x}_1 = 4m$$

Consider Moment $A_2 = 6M_B$, $x_2 = 3m$

Consider 75 kN Load

$$A_3 = \frac{1}{2} \times 300 \times 4 = 600 \text{ kNm}^2$$

$$\bar{x} = 4.667$$

Consider 100kN load.

$A_2 = \frac{1}{2} \times 200 \times 2 = 200 \, kNm^2$

$\bar{x} = 5.333$

$\Sigma_A = 0$

$18 R_B - 6 M_B - 600 - 200 = 0.$

$18 R_B - 6 M_B = 800$

$\Sigma_{Ax} = 0$

$18 R_B \times 4 - 6 M_B \times 3 - 600 \times 4.667$

$- 200 \times 5.333 = 0$

$72 R_B - 18 M_B = 8866.66$

$R_B = 81.48 \, kN$

$M_B = 111.11 \, kNm$

$R_A = 100 + 75 - R_B$

$= 93.52 \, kN$

$81.48 \times 6 - 75 \times 4 - 100 \times 2 - 111.11 = 0$

$M_B = 122.23 \, kNm$

$M_c = 81.48 \times 4 - 75 \times 2 - 111.11$

$= 64.81 \, kNm$

$M_D = 81.48 \times 2 - 111.11$

$= 51.85 \, kNm.$

a.

Consider point B

Left of B = 0

Right of B = 81.48

Left of B = 81.48

Right of D = 81.48 − 75

= 6.48 kNm.

Left of C = 6.48 kNm

Right of C = −93.52 kNm

Left of A = −93.52 kNm.

Right of A = 0 kNm.

1

Unit :- III Column and Cylinders.

Column and Strut

A member of structure (or) base which carries an axial compressive load is called strut. If the strut is vertical (or) inclined at 90° to the horizontal is known as column (or) pillar (or) stanchion.

Slenderness ratio :- k or λ

It is the ratio of unsupported length (or) equivalent of the column to minimum radius of gyration of the section

$$k = \frac{\text{Equivalent length of Column}}{\text{Minimum radius of gyration}}.$$

No Units.

Buckling Load :- (or) Crippling Load :-

The maximum limiting load at which the column tends to have lateral displacement (or) tends to buckle is called buckling (or) crippling load.

Classification of Columns :-

Depend on the slenderness ratio (or) l/d ratio column are divided into 3 types.

1.) Short column 2) Medium column.

3) Long Columns

↳ $l = 8d$ to $30d$.

$K = 32$ to 120

Short columns :-

Column which have length less than 8 times of respective diameter (or) Slenderness ratio less than 32 is called short column.

Long Column :-

The column having their length more than 30 time their respective diameter. (or) Slenderness ratio more than 120 are called long columns.

$l < 30$ time diameter.

$K > 120$

Effective length (or) Equivalent Length :-

The distance between adjacent of inflexion is called equivalent length (or) effective length (or) simple column length.

End condition of the column.

(i) Both end is pin jointed (or) hinged (or) rounder (or) free

Eff. Length (L_e) = Actual Length.

2.

(ii) One end fixed and other end free.

$l_e = 2L$

(iii) One end fixed and other end pin jointed.

$l_e = l/\sqrt{2}$

(iv) Both end fixed

$l_e = l/2.$

Euler's Column theory:-

Assumption :-

⇒ Column is initially straight and uniform lateral dimensions.

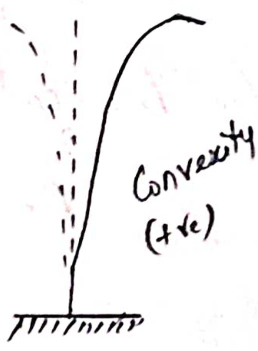
⇒ The weight of the column is neglected

⇒ The column fail by buckling alone.

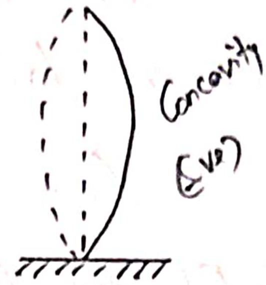
⇒ Compressive Load is exactly axial and it passes through the centroid of the column section.

Sign Convention:-

A moment which trend to bend the column with convexity toward it's initial central line as shown in figure is takes as positive.



A moment which tend to bend the column with concavity forward it's initial central line as shown in figure is taken as negative



Euler's Formula:

Euler's Formula is used for calculating the critical load for a column (or) strut.

$$P_{Euler} = \frac{\pi^2 EI}{l_e^2}$$

where,

$P \rightarrow$ Critical Load.

$E \rightarrow$ Modulus of Elasticity.

$I \rightarrow$ Least Moment of Inertia of a section.

$l_e \rightarrow$ Equivalent length of column.

Limitation:

⇒ It is applicable to an ideal strut only.

⇒ It takes no account of direct stress. It mean that it may give a buckling load for struts far in excess of load which they can withstand under direct compression

Critical Load for prismatic column with different end condition

(a) when both end of the columns are hinged (or) pinned.

$$P = \frac{\pi^2 EI}{l^2}$$

3.

(b) When one end is fixed and other end is free.

$$P = \frac{\pi^2 EI}{4l^2}$$

(c) When one end of the column is fixed and other end pinned (or) hinged.

$$P = \frac{2\pi^2 EI}{l^2}$$

(d) When both end of the column are fixed.

$$P = \frac{4\pi^2 EI}{l^2}$$

Rankine - Gordon formula :-

$$P_{Rankine} = \frac{\sigma_c A}{1+a\left(\frac{l_e}{K}\right)^2}$$

Where

a = Empirical Constant $= \frac{\sigma_c}{\pi^2 E}$

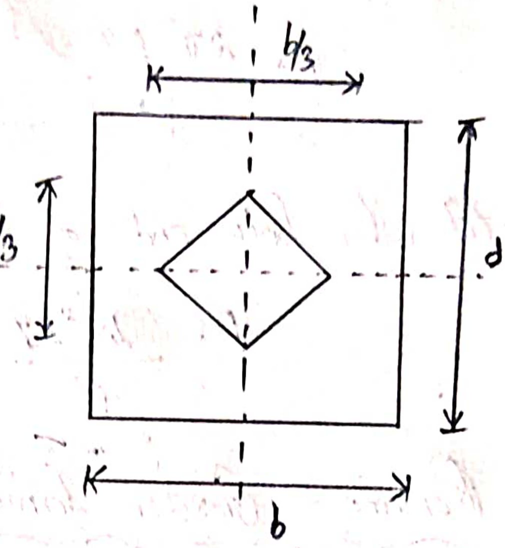
σ_c = Maximum Compressive stress.

A = Sectional area.

l_e = Effective Length.

Core of a section:-

The Load act on the any where in the section and there is no tension is known as core (or) keen of the section.



In a rectangular section.

Section modulus (Z)

$$Z = \frac{1}{6} \times b \times d^2$$

Area, $A = b \times d$

No tension condition, $e \leq \frac{Z}{A}$

$$e \leq \frac{\frac{1}{6} bd^2}{bd}$$

$e \leq \frac{d}{6}$ for rectangular section.

Middle third rule:-

$$e_x \leq b/6$$

$$e_y \leq d/6$$

Tensile stress should not occurs eccentric load acts at any of the geometrical axes. Let it be d/6 it lies in the middle third rule. The stress will be compressive.

The value of eccentricity of e_y on either side of x - axis does not exceed d/6.

4.

Eccentrically Loaded Column :-

Problem:- From the following data of a column of circular section, calculate extreme stresses on the column section. Also find maximum eccentricity in order that there may be no tension any section. External diameter = 20 cm, Internal Diameter = 16 cm, Length of column = 4 m, Load carried by column = 200 kN. Eccentrically of column load = 2.5 cm (from axis of column)

End condition :- Both end are fixed, $E = 94\ GN/m^2$

Solution :-

Area of column, $A = \pi/4 \times (20^2 - 16^2)$

$= 113.1 \times 10^{-4}\ m^2$

$I = \pi/64\ (20^4 - 16^4)$

$= 4.637 \times 10^{-8}\ m^4$

$e = 2.5\ cm = 2.5 \times 10^{-2}\ m$

End condition :- Both end fixed.

$l_e = l/2 = 4/2 = 2m$

Max. BM

$$M_{max} = Pe \sec \frac{l_e}{2} \sqrt{\frac{P}{EI}}$$

$$M_{max} = 200 \times 10^3 \times \left(2.5 \times 10^{-2} \times \sec\left(\tfrac{2}{2}\right)\right) \times \sqrt{\frac{200 \times 10^3}{94 \times 10^9 \times 4637 \times 10^{-8}}}$$

$= 5.1 \ kNm.$

Maximum Compressive stress.

$$\sigma_{max} = \frac{P}{A} + \frac{M}{Z}$$

$$= \frac{200 \times 10^3}{113.1 \times 10^{-4}} + \frac{5.1 \times 10^3}{463.7 \times 10^{-6}}$$

$$Z = I/y$$

$$= \frac{4637 \times 10^{-8}}{10 \times 10^{-2}}$$

$$= 463.7 \times 10^{-6} \ m^3$$

$$\sigma_{max} = 28.7 \ MN/m^2$$

For no tension (Corresponding to the maximum eccentricity).

$$P/A = M/Z$$

$$P/A = \frac{P.e. \sec \frac{le}{2} \sqrt{P/EI}}{Z}$$

$$\frac{200 \times 10^3}{113.1 \times 10^{-4}} = \frac{200 \times 10^3 \times e \times \sec \frac{2}{2} \times \sqrt{\frac{200 \times 10^3}{94 \times 10^9 \times 4637 \times 10^{-8}}}}{463.7 \times 10^{-6}}$$

$$\frac{200 \times 10^3}{113.1 \times 10^{-4}} = \frac{200 \times 10^3 \times e \times 1.02}{463.7 \times 10^{-6}}$$

$$e = \frac{463.7 \times 10^{-6}}{113.1 \times 10^{-4} \times 1.02}$$

$$= 0.0402 \ m$$

$$\boxed{e = 40.2 \ mm}$$

5

Problem:

A 1.5m long cast iron column has a circular cross section of 50mm diameter. one end of the circular column is fixed in direction and position and the other is free. Taking factor of safety as 3. Calculate the safe load using Rankine Gordon formula. Take yield as 560 Mpa and constant $a = 1/1600$

Given:

$$a = 1/1600 \qquad \sigma_c = 560 \times 10^6 \ N/m^2$$

$$l_e = 2L = 2 \times 1.5 = 3m$$

$$A = 1.963 \times 10^{-3} m^2$$

$$A^2 = \frac{I}{A} = \frac{\frac{\pi d^4}{64}}{\pi d^2/4} = \frac{\pi d^4}{64} \times \frac{4}{\pi d^2}$$

$$= \frac{0.05^2}{16} = 1.5625 \times 10^{-4}$$

$$l_e^2 = 3^2 = 9 \quad \frac{l_e^2}{k^2} = \frac{9}{1.5625 \times 10^{-4}} = 57600$$

$$P_{Rankine} = \frac{\sigma_c \times A}{1 + a \times \frac{l_e^2}{k^2}} = \frac{560 \times 10^6 \times 1.96 \times 10^{-3}}{1 + \frac{1}{1600} \times 57600.}$$

$$\text{Safe load} = \frac{29.72}{F.O.S} = \frac{29.72}{3} = 9.9 \ kN$$

Problem:

Find Euler Crippling Load for a hollow Cylinder steel Column of 38mm external diameter and 25 mm. The length of the column is 2.3m and hinged at both ends. Take E = 205 Gpa Also Determine the Crippling Load by Rankine formula using Constant as 335 kN/mm² and 1/7500.

Given

$$a = \alpha = \frac{1}{7500} \quad , \quad \sigma_c = 335 \times 10^3 \ N/mm^2$$

Solution:

$$P_{Rankine} = \frac{\sigma_c \times A}{1 + a\left(\frac{l_e}{k}\right)^2}$$

$$E = 205 \ Gpa = 2.05 \times 10^5 \ N/mm^2$$

Support Condition = hinged at both ends.

$$l_e = l = 2.3m = 2300 \ mm$$

$$k = \sqrt{I/A} = \sqrt{\frac{44140.11}{278.816}} = 12.58$$

$$\frac{l_e}{k} = \frac{2300}{12.58} = 182.83$$

$$I = \frac{\pi . D^4}{64} = \frac{\pi\left(38^4 - 33^4\right)}{64}$$

$$= 44140.11 \ mm^4$$

$$A = \frac{\pi D^2}{4} = \frac{\pi (D^2 - d^2)}{4}$$

$$= \frac{\pi (38^2 - 33^2)}{4}$$

$$= 278.816 \ mm^2$$

$$P_{Rankine} = \frac{325 \times 10^3 \times 278.816}{1 + \frac{1}{7500} \times (182.83)^2}$$

$$= 17116.5 \ kN$$

$$\text{Crippling Load} = \frac{P \times F.O.S}{P_{(Rankine)}}$$

$$\text{Crippling Load} = P_{Rankine} \times F.O.S$$

$$= 17116.5 \times 3$$

$$= 51349 \ kN$$

Thin Cylinderical and spherical shells.

If the thickness of the wall of the Cylinderical vessel is less than 1/15 to 1/20 of its internal diameter, the cylinderical vessel is known as thin Cylinderical, thin spherical shells.

Problem:- A cylinderical pipe of diameter 1.5 m and thickness 1.5 cm is subjected to an internal fluid pressure of 1.2 N/mm^2

determine

(i) Longitudinal Stress developed in the pipe.

(ii) Circumferential Stress developed in the pipe.

Given:- Internal Fluid pressure, $P = 1.2\ N/mm^2$

$$d = 1.5m \; ; \; t = 1.5\,cm = 1.5 \times 10^{-2} m$$

Solution:-

$$t/d = \frac{1.5 \times 10^{-2}}{1.5} = \frac{1}{100} = 0.01 < \frac{1}{20} = 0.05$$

It is Thin Cylinder.

The Longitudinal stress σ_2 :-

$$\sigma_2 = \frac{P \times d}{4t} = \frac{1.2 \times 1.5}{4 \times 1.5 \times 10^{-2}} = 30\ N/mm^2$$

The Circumferential Stress.

$$\sigma_1 = \frac{Pd}{2t}$$

$$= \frac{1.2 \times 1.5}{2 \times 1.5 \times 10^{-2}}$$

$$= 60\ N/mm^2$$

Thick cylinder:

Thick cylinder are the cylindrical vessel, containing fluid under pressure and whose wall thickness is not small $\boxed{t > \frac{d}{20}}$ the following stress are exist in the thick cylinder.

* Radial pressure.
* Hoop stress & circumferential stress.
* Longitudinal stress.

These stresses are obtained using Lami's theory.

1.) Radial pressure

$$(\sigma_r) \text{ (or) } (P_x) = \frac{b}{r^2} - a$$

2.) Hoop stress.

$$(\sigma_c) \text{ or } (f_x) = \frac{b}{r^2} - a$$

Sign Convention

Hoop stress ⇒ (+ve) at tensile
⇒ (−ve) at compressive.

Radial stress ⇒ (+ve) at compressive
⇒ (−ve) at tensile

3) Longitudinal stress

$$(\sigma_e) \text{ (or) } (P_o) = \left[\frac{r_1}{r_2^2 - r_1^2}\right] P.$$

where r - radius , r_1 - Internal radius.

σ_r - Compressive stress.

σ_c - Tensile stress.

Change in dimension.

$$e_v = \frac{\delta v}{v} \text{ (or) } \frac{\delta d}{d}$$

$$\boxed{e_v = 3e}$$

$$V = \frac{4}{3}\pi r^3 \text{ (or) } \frac{\pi d^3}{6}$$

$$\delta d = \frac{Pd^2}{4tE}\left(1 - 1/m\right)$$

Compound Cylinder:

A Compound Cylinder is defined as one cylinder that is shrunk onto the top of another cylinder. The inner cylinder will than be subjected to Compressive hoop stresses. due to outter cylinder will be in tension due to shrinkage.

Shrinking on stresses:

When the working pressure is now applied to the inner cylinder, the stress in the inner Cylinder will be the algebric sum of the stress due to internal pressure.

8.

Problem: A pipe of 200mm internal diameter and 50mm thickness carries a fluid at a pressure of 10 MN/m². Calculate the maximum and minimum intensities of circumferential stress across the section.

Solution:

Internal $r_1 = \frac{200}{2} = 100mm = 0.1m$

External $r_2 = \frac{(200 + 2 \times 50)}{2} = 150 mm = 0.15m$

$P_1 = 10 MN/m^2$

Lame's eqn,

$\sigma_r = \frac{b}{r^2} - a \rightarrow (1)$; $\sigma_c = \frac{b}{r^2} + a \rightarrow (2)$

@ $r = 0.1m$ $\sigma_r = 10 MN/m^2$

When $r = 0.15m$ $\sigma_r = 0$

Substituting in (i) we get

$$10 = \frac{b}{0.1^2} - a$$

$$0 = \frac{b}{0.15^2} - a$$

Solving the above eqn, we get.

$b = 0.18$ and $a = 8$

@ $r = 0.1m$; $\sigma_r = \frac{0.18}{0.1^2} - 8 = 10 \ MN/m^2$

$$\sigma_c = \left[\frac{0.18}{0.1^2}\right] + 8 = 26 \ MN/m^2$$

$r = 0.15m.$

$$\sigma_c = \left[\frac{0.18}{0.15^2}\right] + 8 = 16 \ MN/m^2$$

Unit: IV :: State of stress in three Dimension.

Stress tensor at a point:-

Total stress of any 3-D element is determine by the following stress compound.

$$\tau_{ij} = \begin{bmatrix} \sigma_x & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_y & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_z \end{bmatrix}$$

Stress compound is given by the group of square matrix of stress the compound of mathematical entity is called stress tensor.

where $\sigma_x, \sigma_y, \sigma_z$ = Normal stress.

$\tau_{xy}, \tau_{yx}, \tau_{zx}, \tau_{zy}, \tau_{yx}, \tau_{yz}$ = Shear stress.

Stress invariants:-

A combination of stress at a point do not change with a orientation of co-ordinate axis are called as stress invarient it Denotes as I_1, I_2, I_3

Volumetric Strain:-

The ratio of change in volume of the elastic body due to the external force to the original volume

$$e_v = \frac{\text{Change in Volume}}{\text{Original Volume}} = \frac{\delta v}{V}$$

principal plane:-

The plane which passes in such a manner that the resultant stresses across them in totally normal stress are known as principal plane.

principal stress:-

The normal stress across the plane are termed as principal stress.

Problem:- At a point in a stressed body the principal stresses are 100 MN/m² (T) & 60 MN/m² (C). Determine the normal stress and the shear stress on a plane inclined at 50° to the axis of at major principal stress. Also Calculate the maximum shear stress at the point.

Given:-

$\sigma_x = 100\ MN/m^2\ (T)$, $\sigma_y = 60\ MN/m^2\ (C)$

$= -60\ MN/m^2$

$\sigma = 50^\circ$

Solution:-

Normal stress , $\sigma_n = \dfrac{\sigma_x + \sigma_y}{2} + \dfrac{\sigma_x - \sigma_y}{2}$

$= \dfrac{100 + (-60)}{2} + \dfrac{100 - (-60)}{2} \times \cos 100^\circ$

$= 6.1\ MN/m^2$

2.

Shear stress: (τ)

$$\tau = \frac{\sigma_x - \sigma_y}{2} \times \sin 2\theta .$$

$$= \frac{100 - (-60)}{2} \sin 2(50)$$

$$\tau = 78.78 \ MN/m^2$$

Maximum shear stress:

$$\tau_{max} = \frac{\sigma_x - \sigma_y}{2} = \frac{100 - (-60)}{2}$$

$$\boxed{\tau_{max} = 80 \ MN/m^2}$$

Problem: At a point in a bracket the stress on two mutually perpendicular plane of are 400 MN/m² (T) and 300 MN/m² (C) The shear stress across these plans. is 200 MN/m². Determine magnitude and direction of principle stress and maximum shear stress.

Given:

$\sigma_x = 400 \ MN/m^2$, $\sigma_y = 300 \ MN/m^2$

$\tau_{xy} = 200 \ MN/m^2$

Principal stress:

$$\sigma = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{400 + 300}{2} \pm \sqrt{\left(\frac{400 - 300}{2}\right)^2 + 200^2}$$

$= 144 \ MN/m^2$

Direction of principal stresses.

$$\tan 2\theta = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}$$

$$= \frac{2 \times 200}{400 - 300}$$

$\tan 2\theta = 4.$

$\theta = 38^\circ$ (or) $\theta = 128^\circ.$

Maximum shear stress :-

$$\tau_{max} = \frac{\sigma_1 - \sigma_2}{2} = \frac{556 - 144}{2} = 206 \ MN/m^2$$

The direction of maximum shear stress with plane.

$\sigma_n = 45^\circ + 38^\circ = 83^\circ.$

Theories of failures.

The principal theories are

1) Maximum principal stress theory.

2.) Maximum principal strain theory.

3) Maximum shear stress theory.

4.) Total strain energy theory.

5) Maximum distortion energy theory.

Maximum principal stress theory:

According to this theory failure will occur when the maximum principal tensile stress (σ_1) in the complex system reaches the value of the maximum stress at the elastic limit (σ_{et}) in simple tension or the minimum principal stress reaches the elastic limit stress (σ_{ec}) in simple compression.

$$\sigma_1 = \sigma_{et} \text{ (Tension)}.$$

$$\sigma_3 = \sigma_{ec} \text{ (compressive)} = \sigma_c$$

$$\sigma_1 \leq \sigma$$

Approximately correct for ordinary cartoon and brittle materials.

Problem: For a metallic body the principal stresses are +35 MN/m² and −95 MN/m². The third principal stress being zero. The elastic limit stress in simple tension as well as in simple compression be equal and is 220 MN/m². Find the factor of safety based on the elastic limit of the criterion of failure for the material is the maximum principal stress theory.

Given:-

$$\sigma_1 = +35\ MN/m^2,\quad \sigma_2 = 0,\quad \sigma_3 = -95\ MN/m^2$$

$$\sigma_1 = \sigma_t$$

Solution:

$$\sigma_1 = \frac{\sigma_{et}}{F.O.S}\ \text{(tension)}$$

$$F.O.S = \frac{\sigma_{et}}{\sigma_1} = \frac{220}{35} = 6.28$$

$$|\sigma_3| = \sigma_c\ \text{(compression)}$$

$$|\sigma_3| = \frac{\sigma_{ec}}{|\sigma_3|} = \frac{220}{|-95|} = 2.316$$

So, the material according to the maximum principal stress theory will fail due to the compressive principal stress

Factor of safety = 2.3

4.

Maximum principal strain theory :-

This theory associated with St. Venant. The theory states that the failure of a material occurs when the principal tensile strain in material reaches the strain at the elastic limit in simple tension (or) when the minimum principal strain reaches the elastic limit strain in simple compress.

Problem:- In a steel member, at a point the major principal stress is 180 MN/m² and the minor principal stress is compressive. If the tensile yield point of the steel is 225 MN/m². Find the value of the minor principal stress at which yielding will commence, according to each of the following criteria of failure.

(1) Maximum shear stress

(2) Maximum total ~~strength~~ strain energy and

(3) Maximum shear strain energy.

Take poison ratio = 0.26.

Given :- $\sigma_1 = 180\ MN/m^2$, $\sigma_e = 225\ MN/m^2$

Solution :-

(1) Maximum shearing stress

$$\sigma_1 - \sigma_3 = \sigma_e$$

$$\sigma_3 = -\sigma_e + \sigma_1$$

$= 180 + (-225)$

$= -45 \, MN/m^2$

$\sigma_3 = 45 \, MN/m^2$ (Compression).

2) Maximum total strain energy.

$$\sigma_1^2 + \sigma_3^2 - \frac{2}{m}\sigma_1\sigma_3 = \sigma_e^2$$

$$180^2 + \sigma_3^2 - 2 \times 0.26 \times 180\,\sigma_3 = 225^2$$

$$\sigma_3^2 - 93.6\,\sigma_3 - 18225 = 0$$

$$\sigma_3 = -96.08 \, MN/m^2 \; ; \; \sigma_3 = 96.08 \, MN/m^2$$

(Compression)

3) Maximum shear strain energy :.

$$(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 = 2\sigma_e^2$$

$$\sigma_1^2 + \sigma_3^2 - \sigma_1\sigma_3 = \sigma_e^2$$

$$180^2 + \sigma_3^2 - 180\,\sigma_3 = 225^2$$

$$\sigma_3^2 - 180\,\sigma_3 - 18225 = 0$$

$$\sigma_3 = -72.25 \, MN/m^2 \; ; \; \sigma_3 = 72.25 \, MN/m^2$$

(Compression)

5

Maximum shear stress theory:-

This theory is also called as guest (or) Tresca's theory.

This theory implies that fail will occur when the maximum shear stress τ_{max} in the complex system reaches the value of maximum shear stress in simple tension at the elastic limit.

$$\tau_{max} = \frac{\sigma_1 - \sigma_3}{2} = \frac{\sigma_{et}}{2} \text{ (in simple tension).}$$

$$\sigma_1 - \sigma_3 = \sigma_{et}.$$

This theory has been found give quite. this theory satisfactory result for ductile material.

problem :- A mild steel shaft 120 mm diameter is subjected to a maximum torque of 20 kNm and a maximum bending moment of 12 kNm at a particular section. Find the factor of safety according to the maximum shear stress theory if the elastic limit in simple tension is 220 MN/m^2.

Given:- Diameter of the mild steel shaft, $d = 120$ mm, $d = 0.12$ m

Maximum torque (T) = 20 kNm

Maximum bending moment (M) = 12 kNm / σ_{et} = 220 MN/m^2

Solution :

$$\sigma_b = \frac{32M}{\pi d^3} = \frac{32 \times 12 \times 10^3}{\pi \times 0.12^3}$$

$$= 70.74 \times 10^6 \ N/m$$

$$= 70.74 \ MN/m$$

$$\tau = \frac{16T}{\pi d^3} = \frac{16 \times 20 \times 10^3}{\pi \times 0.12^3} = 58.95 \times 10^6 \ N/m$$

$$= 58.95 \ MN/m$$

Principal stress are given by

$$\sigma = \frac{\sigma_b}{2} \pm \sqrt{\left(\frac{\sigma_b}{2}\right)^2 + \tau^2}$$

$$= \frac{70.74}{2} \pm \sqrt{\left(\frac{70.74}{2}\right)^2 + 58.95^2}$$

$$\sigma = 35.57 \pm 68.75$$

$$\sigma_1 = 35.57 + 68.75 = 104.12 \ MN/m$$

$$\sigma_2 = 35.57 - 68.75 = -33.38 \ MN/m$$

According to the maximum shear stress theory

$$\sigma_1 - \sigma_3 = \sigma_t$$

$$\sigma_1 = 104.12 \ MN/m^2 \ ; \ \sigma_2 = 0$$

$$\sigma_3 = -33.38 \ MN/m^2$$

$\sigma_t = 104.12 - (-33.38)$

$= 137.5 \ MN/m^2$

$$F.O.S = \frac{\sigma_{et}}{\sigma_t} = \frac{220}{137.5} = 1.6$$

Total strain energy theory :-

This theory which has a thermodynamic analogy and a logic basis is due to Haigh.

This theory states that the failure of a material occur when total strain energy theory in material reaches the total strain energy of material at the elastic limit in simple tension.

Maximum distortion energy theory (or) shear strain energy theory

This theory is also called as Mises Henky theory. According to this theory the elastic failure occur where the stress strain energy per unit volume in the stressed material reaches a value equal to the shear strain energy per unit volume at elastic limit point in simple tension.

$$(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1) = \sigma_{et}^2$$

The above theory has been give best result for ductile material for which $\sigma_{et} = \sigma_{ec}$ approximately

Problem: In a material, the principal stresses are $60\ MN/m^2$, $48\ MN/m^2$ and $-36\ MN/m^2$. Calculate (i) Total strain energy (ii) Volumetric strain energy (iii) Shear strain energy (iv) factor of safety on the total strain energy criterion of the material yield at $12\ MN/m^2$. Take $E = 200\ MN/m^2$ and $1/m = 0.3$.

Solution:-

(i) Total strain energy per unit volume:-

$$= \frac{1}{2E}\left[\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - \left(\frac{2}{m}\left(\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1\right)\right)\right]$$

$$= \frac{1}{2\times 200\times 10^9}\left[60^2 + 48^2 + (-36)^2 - \left[2\times 0.3\left(60\times 48 + 48\times(-36) + (-36\times 60)\right)\right]\right]$$

$$= 1951\ MN/m^2$$

(2) Volumetric strain energy:-

$$= \frac{1}{3}\left(\sigma_1 + \sigma_2 + \sigma_3\right)^2\left(\frac{1 - 2/m}{2E}\right)$$

$$= \frac{1}{3}\left(60 + 48 + (-36)\right)^2 \times 10^{12}\left[\frac{1 - (2\times 0.3)}{2\times 200\times 10^9}\right]\times 10^{-3}$$

$= 1.728 \ kNm/m^3$

③ Shear strain energy per unit volume:

$$= \frac{1}{12C}\left[(\sigma_1-\sigma_2)^2 + (\sigma_2-\sigma_3)^2 + (\sigma_3-\sigma_1)^2\right]$$

$$C = \frac{E}{2(1+1/m)} = \frac{200 \times 10^9}{2(1+0.3)} = 76.92 \times 10^9 \ N/m^2$$

$$= \frac{1}{12 \times 76.92 \times 10^9} \times 10^{-12} \left[(60-48)^2 + (48+36)^2 + (-36-60)^2\right]$$

$$= 17.78 \ kN/m^2$$

④ Factor of Safety:

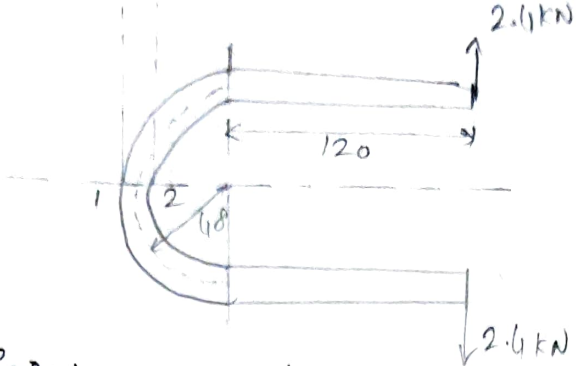
$$\frac{\sigma_e^2}{2E} = \frac{(120 \times 10^6)^2}{2 \times 200 \times 10^9} \times 10^{-3}$$

$$= 36 \ kN/m^2$$

$$F.O.S = \frac{36}{19.51} = 1.845$$

1) Fig shows a frame subjected to a load of 2.4kN. Find (i) the resultant stresses at

direct σ + bend σ ↓ $\frac{L}{A}$



Point 1 and ② (ii) Position of the neutral axis.

1, 2 (σ_d)
Tensile → (+) (+)
Comp → (−) (+)

Sol

R = 48

Resultant stress, $= \sigma_R = \sigma_d + \sigma_b$

direct stress, $\sigma_d = \frac{\text{load}}{\text{Area}}$

$$= \frac{2.4 \times 10^3}{48 \times 18}$$

$$\therefore \sigma_d = 2.77\ N/mm^2$$

$\underline{\sigma_b \text{ at Point } ①}$ (outer edge)

$$\sigma_{b_1} = \frac{M}{AR}\left[1 + \frac{R^2}{h^2}\left(\frac{y}{R+y}\right)\right]$$

$$M = (2.4 \times 10^3) \times (120 + 48)$$

$$\therefore M = -403.2 \times 10^3$$

Degree of curvature bending moment is Negative. Radius of curvature is decreases

$$h^2 = \frac{R^3}{D} \log_e\left(\frac{2R+D}{2R-D}\right) - R^2$$

$$= \frac{48^3}{48} \log_e\left(\frac{2 \times 48 + 48}{(2 \times 48) - 48}\right) - 48^2$$

(ln)

D = 48
$y = \frac{48}{2} = 24$
R = 48

$$= 2304 \times 1.09 - 48^2$$

$$\therefore h^2 = 227.2$$

$A = 48 \times 18 = 864$

$\therefore \sigma_{b_2} \neq M$

$$\sigma_{b_1} = \frac{(-403.2 \times 10^3)}{(864)(48)} \left[1 + \frac{48^2}{227.2} \left(\frac{24}{48+24} \right) \right]$$

$$= -9.72\ (4.38)$$

$\therefore \boxed{\sigma_{b_1} = -42.58\ N/mm^2}$ (comp)

Point ②

$$\sigma_{b_2} = \frac{M}{AR} \left[1 - \frac{R^2}{h^2} \left(\frac{y}{R-y} \right) \right]$$

$$= \frac{(-403.2 \times 10^3)}{(864)(48)} \left[1 - \frac{48^2}{227.2} \left(\frac{24}{48-24} \right) \right]$$

$$= -9.72 \times (-9.14)$$

$\therefore \boxed{\sigma_{b_2} = 88.84\ N/mm^2}$ (tens)

15/3/18 Resultant stress

$\sigma_R = \sigma_d + \sigma_{b_1}$ (Inner edge)

$= 2.77 - 42.58 \cancel{+ 88.84}$

(at point 1) $\boxed{\sigma_{R_1} = -39.81\ N/mm^2}$ (comp)

(outer edge)

$\sigma_{R_2} = \sigma_d + \sigma_{b_2}$

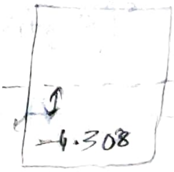
$= 2.77 + 88.86$

$\boxed{\sigma_{R_2} = 91.63\ N/mm^2}$ (tensi)

(ii) Poistion of Neutral axis

$$y = -\frac{h^2 R}{R^2 + h^2}$$

$$y = -\frac{227.2\ (48)}{48^2 + (227.2)}$$

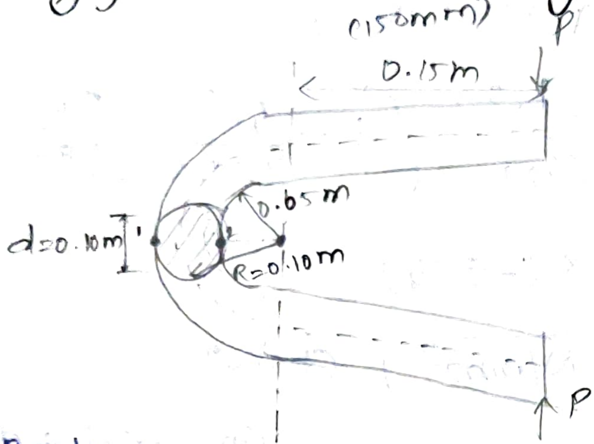


$-\frac{h^2 R}{R^2 + h^2}$

$\therefore y = \cancel{-0.202}\ -4.308$

The distance between (-) symbol means below the centroidal axis.

2) The curved member shown in fig has a solid circular cross section 0.10m in dia. If the maximum tensile and compressive stresses in the member are not to not exceed 150 MPas, 200 MPas respectively. determine the value of load P. that can be safetyly be carried by the member.



Radius of curvature, $R = 0.10m = 100mm$

Dia of cross section, $D = 0.10m = 100\,mm$

$\sigma_1 = 150\,M\,Pascal = 150\,N/mm^2$ (tensile at Point 1)

$\sigma_2 = 200\,M\,pascal = 200\,N/mm^2$ (comp at Point 2)

Pa → N/m²

$\times 10^6 / \times 10^6$

To find safe load P

$$A = \frac{\pi}{4} d^2 = \frac{\pi}{4}(100^2)$$

$$\therefore \boxed{A = 7.854 \times 10^3\,mm^2}$$

In BM

Tensile (−)

comp (+)

Bending moment

$$M = P(150 + 100)$$

(comp) $\therefore \boxed{M = 250P}$

For circular

$$h^2 = \frac{d^2}{16} + \frac{1}{128}\frac{d^4}{R^2} + \ldots$$

$$h^2 = \frac{(100^2)}{16} + \frac{1}{128}\frac{(100^4)}{(100^2)} + \ldots$$

$$= 625 + 78.125$$

$$\therefore \boxed{h^2 = 703.125}$$

Bending stress at Point 1.

$y = \frac{D}{2}$

$= \frac{100}{2}$

$= 50$

$$\sigma_{b1} = \frac{M}{AR}\left[1 + \frac{R^2}{h^2}\left[\frac{y}{R+y}\right]\right] \quad \text{(Tensin)}$$

Tensile (-)

$$= \frac{-P}{7.854\times10^3} + \frac{250P}{(7.854\times10^3)(100)}\left[1 + \frac{(100^2)}{(703.125)}\left(\frac{50}{100+50}\right)\right]$$

$$-1.273\times10^{-4}P + 3.183\times10^{-4}P$$

$$= \cancel{0.031P}\left[1 + 4.740\right]$$

$$\cancel{150}\ \sigma_{b1} = \cancel{0.1117P} \cancel{+1.827\times10^{-3}} P$$

$$\therefore 150 = -1.273\times10^{-4}P + 1.827\times10^{-3}P = \boxed{P = 88.21kN}$$

at Point 2 $150 = 1.699\times10^{-3}P$

$$\sigma_{R2} = \sigma_d + \sigma_{b2}$$

$$200 = \frac{P}{7.85\times10^3} + \frac{250P}{7.85\times10^3\times100}\left(1 + \frac{100^2}{703.125}\left(\frac{50}{100+50}\right)\right)$$

$$200 = (1.273\times10^{-4}\cancel{P}) + 3.183\times10^{-4}P\ \cancel{[5.740]}\ [1-14.22]$$

$$200 = 1.273\times10^{-4}P + \cancel{1.827\times10^{-3}P}\ \cancel{(3.183\times10^{-4}P)}$$

$$= \cancel{2P(1.954\times10^{-3})}\ \cancel{(4.456\times10^{-4})}\ (-4.208\times10^{-3}P)$$

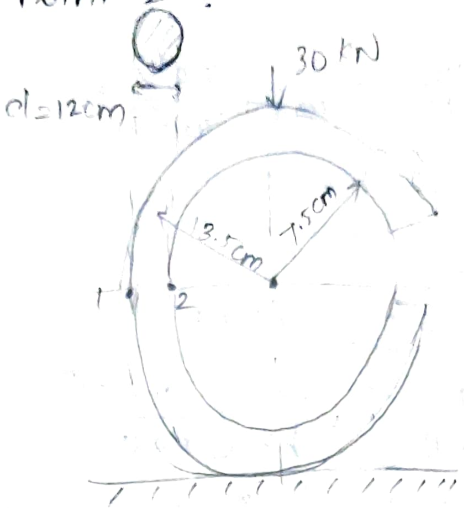
$$= P(-4.681\times10^{-3})$$

$$\therefore P = -48.9kN$$

Result,

safe load $= 48.9kN$ (less value taken)

3) To find the Resultant stresses at Point 1 and Point 2.



<u>Given</u>,

Load $P = 30 \times 10^3 N$

$d = 12cm = 120mm$

$R = 13.5cm = 135mm$

<u>To find</u>. $\sigma_{R_1} = ?$

$\sigma_{R_2} = ?$

<u>soln</u>

$$A = \frac{\pi}{4}(d^2) = \frac{\pi}{4}(120^2)$$

$$A = 11.30 \times 10^3 mm^2$$

$$\sigma_d = P/A = \frac{30 \times 10^3}{(11.30 \times 10^3)} = 2.652 N/mm^2$$

$$h^2 = \frac{d^2}{16} + \frac{1}{128}\frac{d^4}{R^2}$$

$$= \frac{(120^2)}{16} + \frac{1}{128}\frac{(120^4)}{(135^2)}$$

$$\therefore h^2 = 988.888$$

$$BM = (30 \times 10^3) \times 135$$

$$\therefore BM = 4.05 \times 10^6$$

Resultant stress at Point 1

$$\sigma_{R_1} = \sigma_d + \sigma_{b_1}$$

$y = \frac{d}{2} = \frac{120}{2} = 60$

(Tension) (-) ve BM

$$= -P/A + \frac{M}{AR}\left[1 + \frac{R^2}{h^2}\left(\frac{y}{R+y}\right)\right]$$

$$= -2.652 + \frac{(4.05\times10^6)}{(11.30\times10^3)(135)}\left[1 + \frac{135^2}{(988.888)}\left(\frac{60}{135+60}\right)\right]$$

$$= -2.652 + 2.656\ (6.670)$$

$$\therefore \boxed{\sigma_R = 15.05\ N/mm^2}\ (\text{Tensi})$$

Resultant stress at: 2

$$\sigma_{R_2} = \sigma_d + \sigma_{b_2}$$

(Comp) (+) ve BM

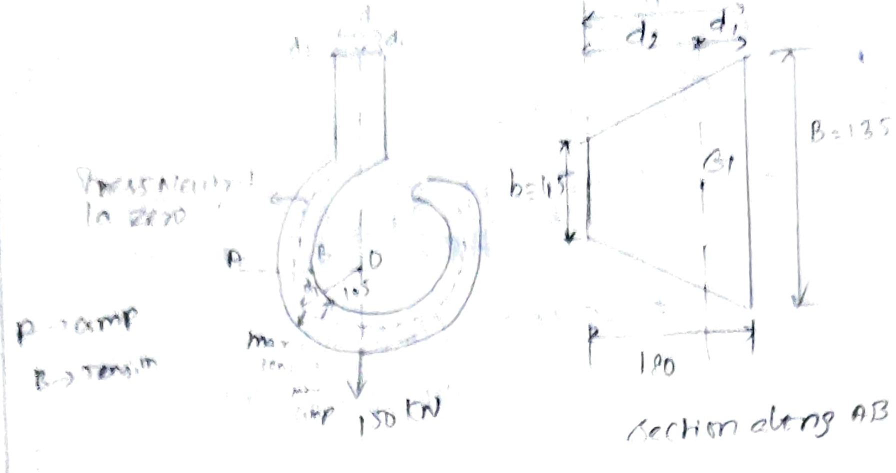
$$= P/A + \frac{M}{AR}\left(1 - \frac{R^2}{h^2}\left[\frac{y}{R-y}\right]\right)$$

$$= 2.652 + \frac{(4.05\times10^6)}{(11.30\times10^3)(135)}\left[1 - \frac{135^2}{988.888}\left(\frac{60}{135-60}\right)\right]$$

$$= 2.652 + (2.656 \times 13.74\cancel{788})$$

$$\boxed{\sigma_{R_2} = -39.1\ N/mm^2}\ (\text{Comp})$$

(4) A fig shows a grain hook ~~hief~~ lifting a load of 150 kN. determine the max compressive & tensile stresses in the critical section of the grain hook.



Given,

$B = 135\,mm$

$b = 45\,mm$

$d = 180\,mm$

$P = 150\,kN = 150\times10^3\,N$

direct both are (+)

To find,
max comp & tensile stresses.

Sol

$A = \frac{1}{2}h(a+b) = \frac{1}{2}180(45+135)$

$\therefore A = 16200\,mm^2$

$d_1 = \frac{d}{3}\left(\frac{B+2b}{B+b}\right)$

$= \frac{180}{3}\left(\frac{135+2\times45}{135+45}\right)$

$\therefore \boxed{d_1 = 75\,mm}$

$d_2 = d - d_1$

$= 180 - 75$

$\therefore \boxed{d_2 = 105\,mm}$

$R = 105 + d_1 = 105 + 75$

$\boxed{R = 180\,mm}$

$$h^2 = \frac{R^3}{A}\left[b\cdot\log_e\left(\frac{R+d_2}{R-d_1}\right) + \left(\frac{B-b}{d}\right)(R+d_2)\log_e\left(\frac{R+d_2}{R-d_1}\right) - (B-b)\right] - R^2$$

$$= \frac{180^3}{16200}\left[45 \log_e\left(\frac{180+105}{180-75}\right) + \left(\frac{135-45}{180}\right)(180+105) \log_e\left(\frac{180+105}{180-75}\right) - (135-45)\right] - (180^2)$$

$$= 360\left[45(0.998) + 142.5(0.998) - 90\right] - 32400$$

$$= 360 \cdot (97.125) - 32400.$$

$$\therefore h^2 = 2565\,mm^2$$

$$\therefore h^2 = 0.002565\,m^2$$

Bending moment.

(Tens)

$$M = -150 \times 10^3 \times 180$$

$$\therefore M = -27 \times 10^6\,Nmm$$

direct both are (-)

Direct stress $\sigma_d = \frac{P}{A} = \frac{150 \times 10^3}{16200} = 9.26\,N/mm^2$

Resultant stress at A.

$$\sigma_A = \sigma_d + \sigma_b$$

$y = D/2 = \frac{180}{2} = 90$

$$= 9.26 + \frac{M}{AR}\left[1 + \frac{R^2}{h^2}\left(\frac{d_2}{R+d_2}\right)\right].$$

$$= 9.26 + \frac{(-27 \times 10^6)}{(16200)(180)}\left[1 + \frac{(180^2)}{(2565)}\left(\frac{105}{180+105}\right)\right]$$

$$= 9.26 + (-9.259)\cancel{(1.001)}(5.653)$$

$$\boxed{\sigma_A = -43.17\,N/mm^2} \text{ (Comp)}$$

at Point B

$$\sigma_B = \sigma_d + \sigma_b$$

$$= 9.26 + \frac{(-27 \times 10^6)}{(16200)(180)}\left[1 - \frac{(180^2)}{2565}\left(\frac{75}{180-75}\right)\right]$$

$$= 9.26 + (-9.259)(-8.022)$$

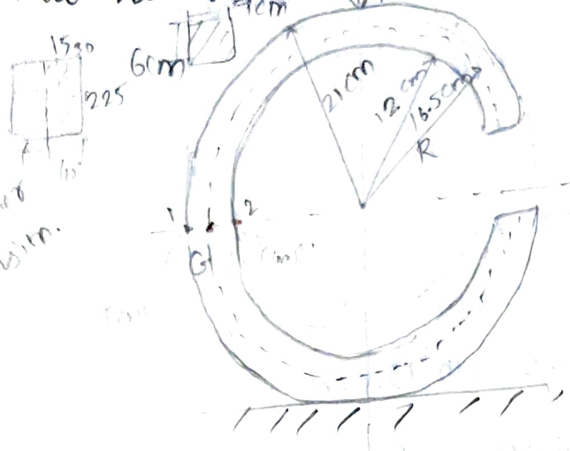
$$\therefore \boxed{\sigma_B = 83.70\,N/mm^2} \text{ (Tension)}$$

16/3/1[illegible]

5) A Fig shows a circular ring of reactangular section, to be slit and subjected to load P. (i) calculate the magnitude of force P if the max stress along the section 1 to 2 is not to exceed 225 MN/m² (comp).

(ii) draw the stress distribution along 1 to 2.

soln



comp is greater than tension.

To find (i) Area of cross section.

Force P.

(ii) Stress distribution dia along sec ① & ②

soln.

(i) Area of cross section $= 90 \times 60 \, mm$

$= 5400 \, mm^2$

Bending moment, $M = P \times 165$

$= +165P$

(ii) Magnitude of Force P.

$\sigma = 225 \, MN/m^2 = 225 \, N/mm^2$

$\sigma_d = P/A$

$= P/5400$

$\therefore \boxed{\sigma_d = 1.851 \times 10^{-4} P}$

σ_b + σ_d should not exceed.

$$h^2 = \frac{R^3}{D}\left[\log_e\left(\frac{2R+D}{2R-D}\right)\right] - R^2$$

$d = 90mm$
$R = 165mm$

$$= \frac{(165^3)}{90}\left[\log_e\left(\frac{2\times165+90}{2\times165-90}\right)\right] - (165^2)$$

$$= 27.931\times10^3 - (165^2)$$

$$\therefore \boxed{h^2 = 706.82 mm^2}$$

(σ_R) $\sigma_2 = 225 MN/m^2 = 225 N/mm^2$

At point 2

$$\sigma_R = \sigma_d + \sigma_{b_2}$$

$$225 = (1.851\times10^{-4})P + \sigma_{b2}$$

$y = D/2$
$= 90/2$

$$225 = 1.851\times10^{-4}P + \frac{M}{AR}\left[1 - \frac{R^2}{h^2}\left(\frac{y}{R-y}\right)\right].$$

$$225 = (1.851\times10^{-4}P) + \frac{(165P)}{(5400)(165)}\left[1 - \frac{(165^2)}{706.82}\left(\frac{45}{165-45}\right)\right]$$

$$225 = (1.851\times10^{-4}P) - 2.489\times10^{-3}P.$$

$$225 = -2.304\times10^{-3}P$$

$$\therefore P = -97.63\times10^3 N \text{ Comp}$$

$$\therefore \boxed{P = -97.6 kN}$$

P value should be in Positive

$$M = 165(+97.6\times10^3)$$

$$\therefore \boxed{M = +16.104\times10^6 Nmm}$$

(ii) Stress distribution diagram.

0-2 direction, $y = 15mm$

$$\sigma_{15} = \sigma_d + \sigma_b$$

$$= P/A + \frac{M}{AR}\left[1 - \frac{R^2}{h^2}\left(\frac{y}{R-y}\right)\right].$$

$$= \frac{(97.6\times10^3)}{5400} + \frac{16.104\times10^6}{5400(165)}\left[1 - \frac{165^2}{706.82}\left(\frac{15}{165-15}\right)\right]$$

$$= \frac{(97.6\times10^3)}{5400} (\cancel{1}\ \cancel{2.8817}\ (-51.529)$$

$$\therefore \sigma_{15} = \cancel{18.071} - 33.48$$

$$\sigma_{30} = \frac{97.6}{5400} + \frac{16.104\times10^6}{5400(165)}\left[1 - \frac{165^2}{706.82}\left(\frac{30}{165-30}\right)\right]$$

$$= \frac{97.6\times10^3}{5400} - (\cancel{136.637}) + (18.074)(-7.559)$$

$$\therefore \sigma_{30} = -\cancel{118.572}$$

$$18.07 - 136.62$$

0-1 direction $\therefore \sigma_{30} = -118.52$

$$\sigma = \sigma_d + \sigma_b$$

$$= -P/A + \frac{M}{AR}\left[1 + \frac{R^2}{h^2}\left(\frac{y}{R+y}\right)\right].$$

$$\sigma_{15} = -\frac{97.6\times10^3}{5400} + \frac{(16.104\times10^6)}{(5400)(165)}\left[1 + \frac{165^2}{706.82}\left(\frac{15}{165+15}\right)\right]$$

$$= -18.074 + 18.074\times(4.209)$$

$$= 57.99\ N/mm^2$$

$$\sigma_{30} = -\frac{97.6\times10^3}{5400} + \frac{(16.104\times10^6)}{(5400)(165)}\left[1 + \frac{165^2}{706.82}\left(\frac{30}{165+30}\right)\right]$$

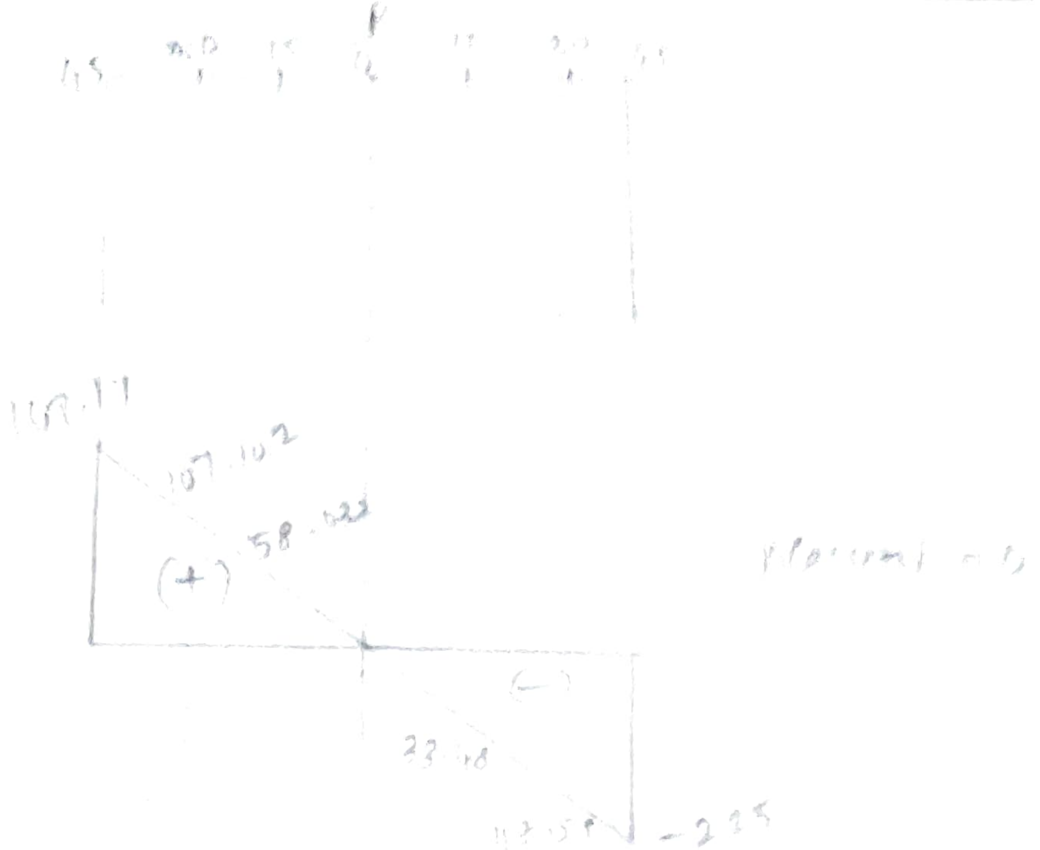
$$= -18.074 + 18.074\times(6.925)$$

$$= 107.102\ N/mm^2$$

$$\sigma_{45} = \frac{-97.6\times10^3}{5400} + \frac{(16.104\times10^6)}{(5400)(165)}\left[1 + \frac{165^2}{706.82}\left(\frac{45}{165+45}\right)\right]$$

$$= -18.074 + 18.074\times(9.253)$$

$$= 149.17\ N/mm^2$$



$y = 0$

$$= \frac{-97.6\times10^{3}}{5400} + \frac{(16.104\times10^{6})}{(5400)(165)}\left[1 \pm \frac{165^{2}}{706.82}\left(\frac{0}{165+0}\right)\right]$$

$$= -18.074 + 18.074 \times 1$$

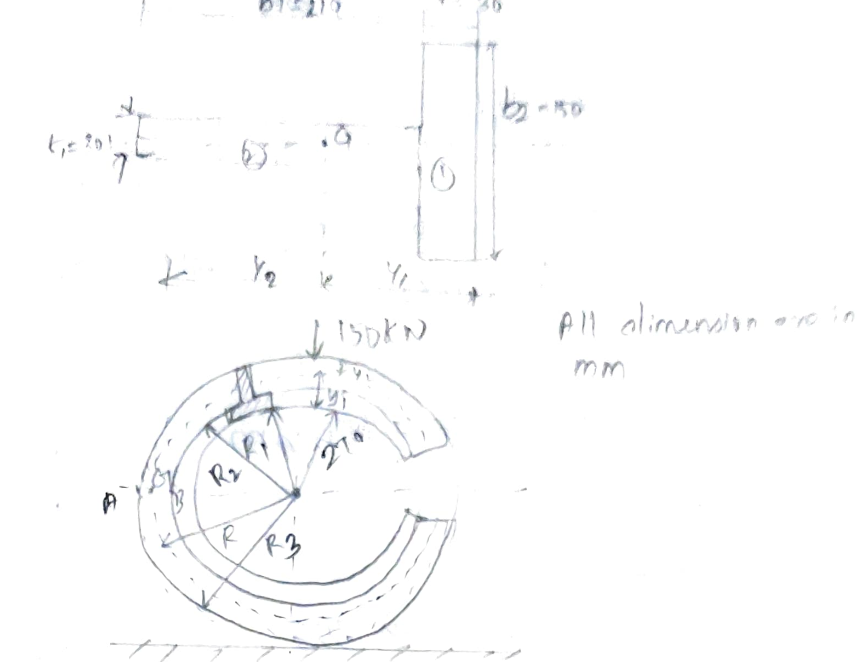
$$= 0$$

centroid neutral axis same for this

since: centroid axis and neutral axis should be coincide.

19/3/18 A fig shows and open ring having T section. Determine the stresses at the points A & B.

If the link is subjected to a load 150 kN.



Soln

To find. σ_{RA}, σ_{RB}

At Point A.

$$\sigma_{RA} = \sigma_d + \sigma_{bn}$$

$$= -P/A + \frac{M}{AR}\left[1 + \frac{R^2}{h^2}\left(\frac{y_2}{R+y_2}\right)\right]$$

$$\sigma_{RB} = \sigma_d + \sigma_{bB}$$

$$= P/A + \frac{M}{AR}\left[1 - \frac{R^2}{h^2}\left(\frac{y_1}{R-y_1}\right)\right]$$

$$\sigma_d = P/A = \frac{150\times10^3}{10.8\times10^3} = 13.89\ N/mm^2$$

$$A = (210\times30) + (150\times30) = 10.8\times10^3\ mm^2$$

$M = P\times R$

$= (150\times10^3)\times$

$R_1 = 270\ mm$

$R_2 = 270+30 = 300\ mm$

$R = ?$

$R = 270 + y_1$

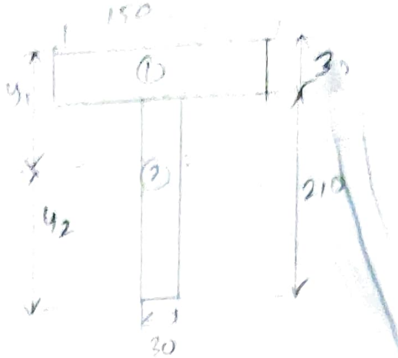
$= 270 + 85$

$= 355\ mm$

$R_3 = 270+30+210$

$= 510\ mm$

To find y_1, y_2



$y_1 = \frac{30}{2} = 15mm$

$y_2 = 30 + \left(\frac{210}{2}\right) = 135mm$

$y_1 = \frac{A_1 y_1 + A_2 y_2}{A_1 + A_2}$

$A_1 = 30 \times 150 = 4500 mm^2$

$= \frac{(4500 \times 15) + (6200 \times 135)}{4500 + 6300}$

$A_2 = 210 \times 30 = 6300 mm^2$

$\therefore y_1 = 85mm$

$\therefore y_2 = 240 - 85 = 155mm$

$t_1 = 30mm$

$b_2 = \cancel{30}\,150 mm$

Moment

$M = P \times R$

$= 150 \times 10^3 \times \cancel{510}\ 355$

$\therefore M = 53.25 \times 10^6$

$R_1 = 270$

$R_2 = 300$

$R = 355$

$R_3 = 510$

$h^2 = \frac{R^3}{A}\left[b_2 \log_e \frac{R_2}{R_1} + t_1 \log_e \frac{R_3}{R_2}\right] - R^2$

$= \frac{(\cancel{510}\,355^3)}{(10.8 \times 10^3)}\left[\cancel{210}\,150 \log_e \frac{300}{270} + 30 \log_e \frac{510}{300}\right] - (355^2)\ \cancel{(300^2)}$

$= \cancel{121.28 \times 10^3}\ \cancel{44.73 \times 10^6}\left[15.804 + 15.918\right] - (355^2)$

$4.142 \times 10^3\ \cancel{44.73 \times 10^6}$

$= \cancel{121.28 \times 10^3} \times (31.722) - (355^2)$

$\therefore \boxed{h^2 = 5.37 \times 10^3 mm^2}$

$\sigma_{RA} = -13.89 + \frac{(53.25 \times 10^6)}{(10.8 \times 10^3)(355)}\left[1 + \frac{355^2}{5.37 \times 10^3}\left(\frac{155}{355 + 155}\right)\right]$

$= -13.89 + (13.88)(8.13)$

$\therefore \boxed{\sigma_{RA} = 99.06\ N/mm^2}$ tensile

$$\sigma_{RB} = 13.87 + \frac{(53.25\times10^6)}{(10.8\times10^3)(355)}\left[1 - \frac{355^2}{5.37\times10^3}\left(\frac{85}{355+85}\right)\right]$$

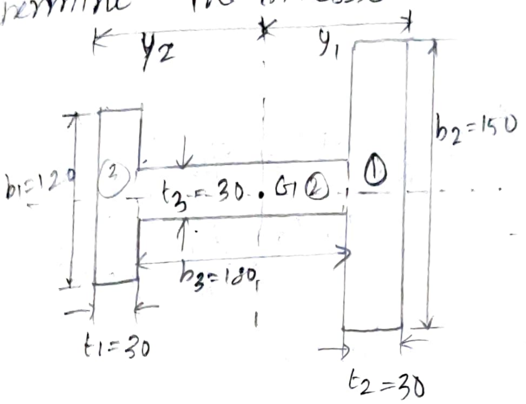
$$= 13.87 + (13.88)(-6.388)$$

$$\boxed{\sigma_{RB} = -102.61\,N/mm^2} \text{ (comp)}$$

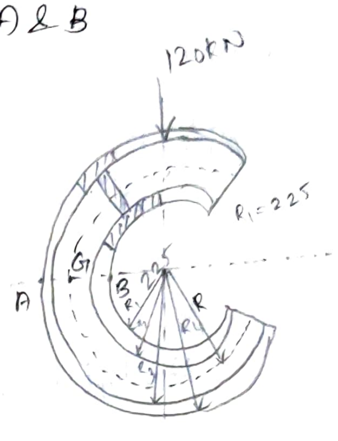
$$\boxed{\sigma_{RB} = -74.775\,N/mm^2} \text{ (comp)}$$

unsymmetrical I section.

shows A C frame subjected to a load of 120 kN determine the stresses at A & B



Section AB



All dimension are in mm

At Point A

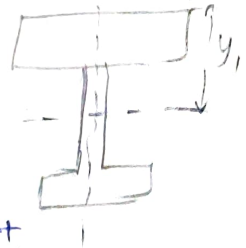
$$\sigma_{RA} = \sigma_d + \sigma_{bA}.$$

$$= -P/A + \frac{M}{AR}\left(1 + \frac{R^2}{h^2}\left(\frac{y_2}{R+y_2}\right)\right)$$

At Point B

$$\sigma_{RB} = \sigma_d + \sigma_{bB}$$

$$= P/A + \frac{M}{AR}\left(1 - \frac{R^2}{h^2}\left(\frac{y_1}{R-y_1}\right)\right).$$



To find y_1, y_2

$$y_1 = \frac{30}{2} = 15\,mm$$

$$y_2 = 30 + \left(\frac{180}{2}\right) = 120\,mm$$

$$y_3 = 30 + 180 + \left(\frac{30}{2}\right)$$

$$= 225\,mm$$

$$y_1' = \frac{A_1y_1 + A_2y_2 + A_3y_3}{A_1 + A_2 + A_3}$$

$$= \frac{(3600\times15) + (4500\times120) + (5400\times225)}{3600 + 4500 + 5400}.$$

$$y_1' = 113\,mm$$

$A_3 = (120\times30) = 3600$

$A_2 = (30\times150) = 4500$

$A_2 = (30\times180) = 5400$

$A = 13500\,mm^2$

$Y_2 = 240 - 113$

$Y_2' = 127 mm$

step: 2

$R = 225 + Y_1 = 225 + 113 = 338 mm$

$R_1 = 225 mm$

$R_2 = 225 + 30 = 255 mm$

$R_3 = 225 + 30 + 180 = 435 mm$

$R_4 = 225 + 30 + 180 + 30 = 465 mm$

step: 3.

$$h^2 = \frac{R^3}{A}\left[b_2 \log_e \frac{R_2}{R_1} + t_3 \log_e \frac{R_3}{R_2} + b_1 \log_e \frac{R_4}{R_3}\right] - R^2$$

$A = b_1 t_1 + b_2 t_2 + b_3 t_3$

$A = (120 \times 30) + (30 \times 150) + (180 \times 30)$

$= 13500$

$$= \frac{338^3}{13500}\left[\cancel{150}\,180 \log_e \frac{255}{225} + 30 \log_e \frac{435}{255} + \cancel{120}\,150 \log_e \frac{465}{\cancel{338}\,435}\right] - (338^2)$$

$= 2.860 \times 10^3 \left[\cancel{0.4}\ 18.776 + 16.022 + 8.002\right] - (338^2)$

$= 2.860 \times 10^3 \times (42.798) - (338^2)$

$h^2 = 8.122 \times 10^3 mm^2$

Step: 4

$$M = P \times R$$
$$= (120\times10^3) \times (338)$$
$$= 40.56\times10^6 \text{ Nmm}$$

Step: 5

$$\sigma_d = P/A = \frac{(120\times10^3)}{13500} = 8.89 \text{ N/mm}^2$$

Step: 6

$$\sigma_{RA} = -8.89 + \frac{40.56\times10^6}{(13500)(338)}\left[1 + \frac{338^2}{8.122\times10^3}\left(\frac{127}{338+127}\right)\right]$$

$$= -8.89 + 8.888\,(4.861)$$

$$\therefore \boxed{\sigma_{RA} = 33.95 \text{ N/mm}^2} \text{ (Tension)}$$

$$\sigma_{RB} = 8.89 + \frac{40.56\times10^6}{13500(338)}\left[1 - \frac{338^2}{8.122\times10^3}\left(\frac{113}{338-113}\right)\right]$$

$$= 8.89 + 8.888\,(-6.064)$$

$$\therefore \boxed{\sigma_{RB} = -45.009} \text{ (comp)}$$

20/03/18

A curved bar is form of a tube of 120mm outside dia and 7.5mm thickness. the center line of this beam is a circular or of radius 225mm. a bending moment of 3kNm bending to (+) increase curvature of the bar is apply. Calculate the max tensile & compressive stresses setup in the bar